Engineering Structures 97 (2015) 90–104

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Design and testing for shape control of piezoelectric structures using topology optimization

Quantian Luo, Liyong Tong *

School of Aerospace, Mechanical and Mechatronic Engineering, The University of Sydney, Sydney, NSW 2006, Australia

ARTICLE INFO

Article history:
Received 11 July 2014
Revised 1 April 2015
Accepted 1 April 2015
Available online 20 April 2015

Keywords:
Shape control
Optimization
Mutual strain energy
Moving iso-surface threshold

ABSTRACT

This paper presents a numerical and experimental investigation into optimum topological design of morphing piezoelectric structures using a moving iso-surface threshold method. Proposed first is a novel formulation of the response function suitable for minimizing the error norm between desired and achieved shapes in the form of combined mutual strain energy densities. A design updating algorithm with an enhanced efficiency is then developed in searching for the optimum iso-surface threshold during iterations. Numerical results are presented for optimum topological material distribution for shape morphing of piezoelectric plates subjected to mechanical loading, one channel electrical voltage and a combination of both mechanical and electrical loadings. Experiments for selected optimal topological designs are conducted to validate the present response function and updating algorithm as well as the numerical results.

1. Introduction

Piezoelectric materials and structures have been widely studied due to the coupling effects between electrical and mechanical fields. Flexible structures with piezoelectric actuators and sensors have found a wide range of applications including structural vibration control, structural health monitoring and structural shape control. Shape control or morphing represents an important application [1–4] and there is a growing interest in morphing flexible structures using piezoelectric actuators and sensors.

A desired structural shape of piezoelectric (PZT) structures may be achieved by optimal input voltages [1,3,5], optimal designs of piezoelectric actuators [6–8] and material distributions [9,10]. Finite element based topology optimization is useful for shape morphing to acquire better structural performance as it can be used to optimize voltage input, piezoelectric material distribution and structural material topology [4,11–13]. In this paper, optimal designs of structural materials for morphing piezoelectric structures will be investigated by using a moving iso-surface threshold method (MIST).

Finite element based topology optimization has been substantially studied and many methods have been proposed. A homogenization method [14] is to obtain optimal structural design by homogenizing anisotropic composite with micro-scale voids. In the solid isotropic material with penalization method (SIMP) [15–17], only one variable (0 or 1 in a white-black design) is required to describe one element. In the evolutionary structural optimization method (ESO) [18,19], elements with lower physical responses are gradually removed. In the level set method (LSM) [20–22], design boundary is implicitly expressed using the zero level set of a higher-order surface function. In the moving iso-surface threshold method (MIST) [23,24], a physical response for an objective function is used; each element is depicted using one variable and a moving iso-surface threshold is used to evolve design boundary. That is, features of ESO, SIMP and LSM methods are integrated into MIST. This method has been used to maximize structural stiffness and to design gripping mechanisms and pressurized actuators [23,24]. It will be further studied and used to investigate shape morphing for piezoelectric structures in this paper.

A displacement component at one point can be obtained by applying a unit dummy force based on the principle of virtual work. By using this principle, optimization to design compliant mechanism has been studied by many authors [23,25,26]. In the mechanism design, an objective function can be defined as a difference between the target and realized displacements at one point where a flexible spring is often used.

In the present investigation, structural displacements are firstly expressed by using mutual strain energy and the plate shapes are represented by using deflections at observation points. An objective function is defined by a sum of squared errors between the

* Corresponding author. Tel.: +61 2 93516949; fax: +61 2 93514841.
E-mail address: liyong.tong@sydney.edu.au (L. Tong).

http://dx.doi.org/10.1016/j.engstruct.2015.04.006
0141-0296/

desired and computed deflections at these points. A novel formulation of a response function is derived on the basis of mutual strain energy density and is then used to find optimal topologies of structural and piezoelectric materials as well as voltages for morphing PZT structures. A novel algorithm is also developed to efficiently find an iso-surface level in an iterative process.

Numerical results computed by using MIST are presented for morphing bending and twisting shapes of plates subjected to mechanical and/or electrical loadings. A combination of the two typical shapes can comprise different shapes and thus they are used to show effectiveness of the present algorithm. Specimens for the obtained optimal designs are fabricated and experimental tests are conducted to verify the present computations. Overshoot issues caused by applied voltages in piezoelectric structures are experimentally investigated, which are important to achieve precise shape control in practice.

2. Problem statement and formulation

2.1. Basic problem statement

Topology optimization for shape morphing of piezoelectric structures can be described as [8,12,27] to find the optimal material distribution that minimizes the selected shape error norm E_r subject to relevant equilibrium equations for PZT structures and material volume constraint. In addition, constraints on electric energy consumption or electric field strength should also be introduced [8,12,28]. The shape morphing problem is to minimize an objective function defined as a sum of squared errors between the desired and achieved shapes [1,3,12]:

$$\text{Minimize}: \quad E_r = \sum_{i=1}^{n}(u_i - u_{di})^2 \tag{1a}$$

$$\text{Subject to}: \quad [\boldsymbol{K_{uu}}]\{\boldsymbol{U_k}\} = \{\boldsymbol{F_k}\} - [\boldsymbol{K_{u\varphi}}]\{\boldsymbol{V}\} \tag{1b}$$

$$[\boldsymbol{K_{uu}}]\{\boldsymbol{U_i}\} = \{\boldsymbol{F_i}\} \tag{1c}$$

$$\sum_{e=1}^{N_e}(x_e v_e) \leqslant v_f v_0 \tag{1d}$$

$$\max\{V_e\} \leqslant V_{\max} \tag{1e}$$

$$0 < x_{\min} \leqslant x_e \leqslant 1 \tag{1f}$$

where u_i and u_{di} denote the achieved (or computed) and desired (or specified) displacements at point i; $[\boldsymbol{K_{uu}}]$ and $[\boldsymbol{K_{u\varphi}}](= [\boldsymbol{K_{\varphi u}}]^T)$ are the structural stiffness and piezoelectric coupling matrices; $\{\boldsymbol{F_k}\}$ and $\{\boldsymbol{F_i}\}$ denote the applied and unit dummy force vectors; $\{\boldsymbol{U_k}\}$ and $\{\boldsymbol{U_i}\}$ are the displacement vectors due to the applied loads and the unit dummy force; $\{\boldsymbol{V}\}$ is the voltage vector; V_e ($e = 1,2,\ldots,N_e$) is the voltage of the eth PZT element and N_e is the total element number; $V_{\max}$ denotes the maximum voltage; v_e and v_0 denote the volume of an element and entire design domain; v_f denote the prescribed volume fraction; $\{\boldsymbol{x_e}\}$ is the design variable vector and x_e ranges from 0 to 1; $x_{\min}$ (= 10^{-3} normally) is the minimum value of the design variables. In a black–white design, $x_e = 1$ or $\to 0$.

Eq. (1b) is a simplified equation for piezoelectric structures with converse piezoelectric effect only. When direct and converse piezoelectric effects are considered, equilibrium equations in finite element analysis can be expressed as [1]:

$$\begin{bmatrix} \boldsymbol{K_{uu}} & \boldsymbol{K_{u\varphi}} \\ \boldsymbol{K_{u\varphi}^T} & \boldsymbol{K_{\varphi\varphi}} \end{bmatrix} \begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{V} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F} \\ \boldsymbol{Q} \end{Bmatrix} \tag{2a}$$

where $[\boldsymbol{K_{\varphi\varphi}}]$ is the dielectric matrix and $\{\boldsymbol{Q}\}$ denotes the electric charges. In finite element analysis for piezoelectric structures, the actuation equation can be obtained by [1]:

$$\{\boldsymbol{U}\} = [\overline{\boldsymbol{K}}_{\boldsymbol{uu}}]^{-1}\{\overline{\boldsymbol{F}}\} \tag{2b}$$

where

$$[\overline{\boldsymbol{K}}_{\boldsymbol{uu}}] = [\boldsymbol{K_{uu}}] - [\boldsymbol{K_{u\varphi}}][\boldsymbol{K_{\varphi\varphi}}]^{-1}[\boldsymbol{K_{u\varphi}^T}] \tag{2c}$$

$$[\overline{\boldsymbol{F}}] = \{\boldsymbol{F}\} - [\boldsymbol{K_{u\varphi}}][\boldsymbol{K_{\varphi\varphi}}]^{-1}\{\boldsymbol{Q}\} \tag{2d}$$

If the electro-mechanical fields are weakly coupled, which is true for many commercially-available piezoelectric materials, the actuation equation can be simplified as [1]:

$$\{\boldsymbol{U}\} = [\boldsymbol{K_{uu}}]^{-1}(\{\boldsymbol{F}\} - [\boldsymbol{K_{u\varphi}}]\{\boldsymbol{V}\}) \tag{2e}$$

In shape morphing for piezoelectric structures, this equation is normally adopted [1,2,29] and will also be used in this paper,

When Eq. (1) is used in topology optimization, the structural stiffness and piezoelectric coupling matrices in an iterative process are calculated by [8,12]:

$$[\boldsymbol{K_{uu}}] = \sum_{e=1}^{N_e}\left(x_e^p[\boldsymbol{k_e^{str}}] + x_e^q[\boldsymbol{k_e^{pzt}}]\right) \tag{3a}$$

$$[\boldsymbol{K_{u\varphi}}] = \sum_{e=1}^{N_e}x_e^r[\boldsymbol{k_{(u\varphi)e}}] \tag{3b}$$

in which $[\boldsymbol{k_{(u\varphi)e}}]$ is the piezoelectric coupling matrix; $[\boldsymbol{k_e^{str}}]$ and $[\boldsymbol{k_e^{pzt}}]$ are the stiffness matrices for host structural and piezoelectric materials with solid materials; subscript e represents the eth element; p, q and r are the penalty parameters. p is the penalty of structural materials; q and r are the penalty parameters for the elastic and piezoelectric constants of piezoelectric materials. In a two-dimensional finite element analysis, penalties p and q can be chosen as [8,12]: $p = 3$ or $p = 0$; $q = 3$ or $q = 0$; $r = 1$. For PZT laminated plates, if $p = 3$ and $q = 3$, topology of the PZT material is the same as that of the host structural material and the host structural materials or the PZT materials will not be removed when $p = 0$ or $q = 0$.

Morphing PZT structures can be achieved by optimal voltage channel designs [1,3,28] with an energy consumption and/or maximum voltage constraint. As indicated in [12], an arbitrary spatially distributed voltage field for the PZT actuators may be hardly realized as an optimal voltage distribution usually demands a complicated electric channel configuration so that it is too expensive to implement. Therefore, one channel actuation voltage was used in [12]. In this case, the maximum voltage constraint in Eq. (1e) is used. In this paper, one channel voltage input is also considered; topology optimization for morphing piezoelectric structures will be investigated by using MIST where an objective function is derived on the basis of mutual energy density and the numerical results with experimental validation are presented.

2.2. MIST formulation

MIST [23,24] comprises of three key points: (1) expressing both objective function and constraint in the form of an integral over the design domain, (2) selecting the response (Φ) function, and (3) searching for optimal solutions for a sequential approximate optimization formulations based on structural responses of previous approximations.