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High cycle fatigue simulation: A new stepwise load-advancing strategy

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ABSTRACT

A stepwise load-advancing strategy for cyclic loading will be presented in this paper that yields convergence in reasonable computational time for highly nonlinear behaviour occurring past the *S–N* curve. The algorithm is also effective when dealing with combinations of cyclical loads. The strategy is coupled to a continuum damage model for mechanical fatigue analysis. A brief overview of the constitutive model is also presented although it is not the main focus of this work. The capabilities of the proposed procedure are shown in two numerical examples. The model is validated by comparison to experimental results.

1. Introduction

Steel fatigue has been extensively studied at microstructural level with a clear emphasis on its chemical structure and on the influence that the latter has on material behaviour and its failure. When looking at the phenomenon from the microscale, it can be seen that a large amount of the material internal energy is spent in a rearrangement of its internal structure to accommodate better the cyclical load, followed by the gliding of the interatomic planes phase. Metal grains suffer plastic slip and non-linear behaviour [1], and these irreversible processes are responsible for crack initiation under cyclic loading.

Regarding the high cycle fatigue (HCF) phenomenon, it is known that the type of fracture involved at macroscale level occurs with little or no plastic deformation [2]. Therefore, HCF does not introduce macroscopic plastic strain, but it introduces porosity [3]. These are the reasons that have led to describe this failure mode by means of damage models. These can be categorized into five groups: damage curve approach, crack growth-based approach, life curve modification approach, energy based damage theories and continuum damage mechanics (CDM) approaches [4]. However, in spite of the great number of models proposed in the HCF field, there is not yet a universally accepted one.

In particular, the CDM approach is based on the original concepts of Kachanov [5,6] for treating creep damage problems. The posterior work of Chaboche [7,8], Chaboche and Lemaitre [9,10], Wang [11], Wang and Lou [12], Li et al. [13] and Oller et al. [14] established the CDM framework as a valid alternative to the fracture mechanics formulations in order to asses in a unified way both crack initiation and propagation. Furthermore, they enhanced the study of fatigue problems by recognizing that the theoretical structure of continuum mechanics, such as damage, is suitable for the study of nonlinear fatigue problems and that the mechanical effect known as fatigue produces a loss of material strength as a function of the number of cycles, load amplitude, and reversion index.

Regarding fatigue life prediction, many different approaches have been proposed such as the early methods of stress-life approach and strain-life approach [15]. One of the most used models is based on the Palmgren–Miner linear damage law [16,17]. However, such models do not recognize the effects of prior history of loading, or the load sequence on the subsequent life. Strain-life models, on the other hand, account for the local plasticity effects at stress concentrations regions [18]. Information is abundant in literature as there are many different crack initiation models [19], with a large number of empirical models proposed for the long crack growth prediction [20]. Despite the abundant information existent on fatigue constitutive models, no attention is given to load advancing strategies utilized in numerical simulations, where the main focus of this paper resides.

The basis of the fatigue constitutive model used was initially developed by Oller et al. [14]. The model establishes a relationship between the residual material strength and the damage threshold evolution, controlled by the material internal variables and by a new state variable of fatigue that incorporates the influence of the cyclic load. A brief overview of the constitutive formulation for the HCF case is provided in order to clarify the material behaviour exhibited in the numerical examples. Several model assumptions are to be made. Defect concentration on the microscale

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occurs during the whole period of cyclic loading. This is reflected in the model in a continuous reduction of material strength, occurring even in the elastic stage. Stiffness degradation occurs only in the post critical stage, once the *S*–*N* curve has been passed and, therefore, only in the final stage before failure. The damage parameter has a phenomenological significance indicating the irreversibility of the fatigue process.

Depending on the size of the domain chosen for the fatigue numerical simulation, computational time can vary considerably. Nowadays, running simulations at macroscale level (mechanical part, structural element) continues to be a challenge, especially if the high level of structural complexity attained at the microscale needs to be taken into account to some extent at other scales. This paper aims at offering a stepwise load-advancing strategy that allows a saving of computational time and can help push the barrier of what if possible in terms of numerical simulation one step further.

The strategy can be especially effective when dealing with HCF where material lives are in the range of 10^6–10^7 cycles. If a single loading cycle is described by n loading steps, then the number of loading steps required to complete a HCF analysis would be in the order of $10^7 \times n$. Furthermore, if the mechanical piece has a complex geometry and a high level of discretization is required at finite element level, then at each of the $10^7 \times n$ load steps a large number of constitutive operations need to be computed for each integration point. The above serve as a clear example of why load-advancing strategies are of the utmost importance in HCF simulations.

Furthermore, increasingly more attention has been given to material behaviour in the very high cycle regime from an experimental point of view. The general belief that steel experiences no alteration in its properties after reaching its fatigue limit at 10^7 cycles has been invalidated [21–23]. In this context, this paper provides a tool for rapid automatized time-advance that allows taking numerical simulations beyond the limit of 10^7 cycles in reasonable computational time.

2. Fatigue damage model

A brief description of the constitutive model used in this paper is offered in this section. The fundamentals of a fatigue continuum damage model are presented with a clear emphasis on the model's dependence on *S*–*N* curves. An exhaustive description of the formulation used can be found in Oller et al. [14], where the complete thermo-mechanical constitutive model for the prediction of fatigue effects in structures is formulated. The model is capable of taking into account the combined effect of mean stress and multi-axial stress states. The treatment of the highly complex processes generated by fatigue is made from a phenomenological point of view.

2.1. Mechanical damage formulation

The free Helmholtz energy is formulated in the reference configuration for elastic Green strains, $E_{ij} = E^e_{ij}$, as [24,25]

$$\Psi = \Psi(E_{ij}, d) = (1-d)\frac{1}{2m^0}(E_{ij}C^0_{ijkl}E_{kl}) \tag{1}$$

where m^0 is the material density, $E_{ij} = E^e_{ij}$ is the total strain tensor, $0 \leqslant d \leqslant 1$ is the internal damage variable taking values between its initial value 0 and its maximum value 1 and C^0_{ijkl} is the original constitutive tensor.

Considering the second thermodynamic law (Clausius–Duhem inequality – [26–28]), the mechanical dissipation can be obtained as [24]

$$\Xi = -\frac{\partial \Psi}{\partial d}\dot{d} \geqslant 0 \tag{2}$$

The accomplishment of this dissipation condition (Eq. (2) demands that the expression of the stress should be defined as (Coleman method; see [28])

$$S_{ij} = m^0 \frac{\partial \Psi}{\partial E_{ij}} = (1-d)C^0_{ijkl}E_{kl} \tag{3}$$

Also, from the last expressions, the secant constitutive tensor C^s_{ijkl} can be obtained as:

$$C^s_{ijkl}(d) = \frac{\partial S_{ij}}{\partial E_{ij}} = m^0 \frac{\partial^2 \Psi}{\partial E_{ij}\partial E_{kl}} = (1-d)C^0_{ijkl} \tag{4}$$

where S_{ij} is the stress tensor for a single material point.

2.2. Threshold damage function oriented to fatigue analysis. Phenomenological approach

The effects caused by applying an increasing number of loading cycles are taken into account by means of a proposed $f_{red}(N, S_{\max}, R)$ function. This function is introduced in the above formulation in the expression of the damage threshold surface, $F^D(S_{ij}, d)$, proposed by [28–30]. The number of cycles N can then be incorporated as a new variable. This enables the classical constitutive damage formulation to account for fatigue phenomena by translating the accumulation of number of cycles into a readjustment and/or movement of the damage threshold function.

The non-linear behaviour caused by fatigue is introduced in this procedure implicitly, by incorporating a fatigue state variable $f_{red}(N, S_{\max}, R)$, that is irreversible and depends on the number of cycles, the maximum value of the equivalent stress in the material $S_{\max}$, and on the factor of reversion of the equivalent stress, $R = \frac{S_{\min}}{S_{\max}}$. This new variable affects the residual strength of the material by modifying the damage threshold, $F^D(S_{ij}, d, N)$, either on the equivalent stress function $f^D(S_{ij})$ (Eq. (5a)), or on the damage strength threshold $\overline{K}^D(S_{ij}, d)$ (Eq. (5b)) [14].

$$F^{D'}(S_{ij}, d, N) = \underbrace{\frac{f^D(S_{ij})}{f_{red}(N, S_{\max}, R)}}_{f^{D'}(S_{ij}, N, R)} - \overline{K}^D(S_{ij}, d) \leqslant 0 \tag{5a}$$

$$F^{D''}(S_{ij}, d, N) = f^D(S_{ij}) - \underbrace{\overline{K}^D(S_{ij}, d) \cdot f_{red}(N, S_{\max}, R)}_{K^{D'}(S_{ij}, d, N)} \leqslant 0 \tag{5b}$$

In the above, $f^{D'} = f^D / f_{red}(N, S_{\max}, R)$, is the reduced equivalent stress function in the undamaged space, $K^{D'}(S_{ij}, d, N)$ is the fatigue damage strength threshold, and $d = \int_0^t \dot{d}dt$ the damage internal variable. In the following, the form in Eq. (5a) has been used for the damage threshold criterion.

The evolution of the damage variable is defined as

$$\dot{d} = \dot{\mu}\frac{\partial F^D}{\partial f^D} \tag{6}$$

being μ the consistency damage factor, which is equivalent to the consistency plastic factor defined in [24]. Consequently, for the isotropic damage case,

$$\dot{d} = \frac{\dot{\mu}}{f_{red}} \tag{7}$$

2.3. Function of residual strength reduction for fatigue – Wöhler curve definition

Wöhler or "Stress–Num. of cycles" (*S*–*N*) curves (Fig. 1) are experimentally obtained by subjecting identical smooth specimens