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A reduced model for the design of glass structures subjected to dynamic impulse load

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A B S T R A C T

A reduced finite element model for determining the maximum principal stress of a glass pane subjected to dynamic impact load is developed and compared to a full dynamic finite element model. The reduced model is based on the Rayleigh-Ritz method. The Ritz vectors used are determined by simple static load-cases. The model is applicable to centrically and eccentrically applied impact and to glass of various support conditions. It is demonstrated that the model performs well for various types of supported glass panes and impact applied at different locations on the glass pane. The applicability to small or medium sized glass panes is shown through a parametric investigation in which the results are compared to linear and nonlinear finite element solutions. For large glass panes, especially at smaller glass thickness, geometric nonlinear effects must be considered in the analysis. An outline of how to expand the model to include nonlinear geometric effects is given. Finally, it is shown that the reduced model performs excellently in the modeling of a standard laminated glass balustrade with clamp fixings. Apparently, the model is very well suited for strength design of commonly used glass structures.

1. Introduction

When performing strength design of glass structures, dynamic impact load is one of the load cases that often need to be included in the analysis. Since glass is a brittle material, it is sensitive to impact load and it is necessary to accurately determine the stress distribution in the glass due to this load case. Most often, strength design of glass structures subjected to dynamic impact load is performed by means of experimental tests. The European standard EN-12600, [26], is available to classify glass for impact strength. Other authors, [25,11], have confirmed that this test method yields reproducible results and that the test is easy to perform. However, when performing strength design of general glass structures, the properties of the structure may be different from those prescribed in the standard. It is of course possible to use the impactor described in the EN-12600 standard in strength design testing of general glass structures. In [19] several experimental tests of glass panes of various sizes and support conditions subjected to dynamic impact load are performed and data for calibration and development of analytical and numerical models are extracted. The process of experimental testing is, however, time consuming when considering parameter variation in strength design.

An alternative to experimental tests is to use finite element simulations. Several authors have demonstrated the applicability of full transient finite element simulations in order to simulate the application of impact load to glass structures, [21,22,20,23,24,27]. However, finite element modeling of transient impact load is advanced, time consuming and may require access to advanced commercial finite element programs.

In the design process it is important that the design tools used are such that alternative designs may be tested in an interactive fashion. To achieve this, the methods applied must be very time-efficient.

To make the model more computationally efficient the size of the finite element model can be reduced by means of model reduction techniques. Several methods that are variants of the Rayleigh-Ritz procedure, [4], are available. The methods can be subdivided into the following main categories, [9]: generalized coordinate methods, condensation methods and component mode synthesis. When generalized coordinates are introduced, the system is described using only a few deflection shapes of the original system where for example eigenvectors, [4], Lanczos vectors, [6], or Ritz vectors, [2], are utilized. Condensation methods involve removing the degrees of freedom that are not necessary in order to describe the dynamic behavior of the system. This can be accomplished using for example static (Guyan) condensation, [14], or dynamic

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condensation, see for instance [15]. In component mode synthesis, [8], the domain of the problem is divided into subdomains and each subdomain is described by a different set of basis vectors.

Reduced modeling of glass subjected to dynamic impact load was addressed in [22–24,27]. In [22], a two-degree-of-freedom model, [7], was developed and compared to numerical and experimental results. The applicability of the model is limited to four-sided and two-sided supported glass panes. In terms of the above categorization of reduction methods, the method is based on generalized coordinates using Ritz vectors for representation of the glass pane. One Ritz vector is used in the modeling of the glass, i.e. the static deformation mode. It was noted that when the glass panes have larger width than two meters, the use of the static deformation mode of the glass yields an unrealistic representation of the dynamic behavior of the glass.

The contributions in [23,24] develop different formats of the reduced model of [22]. In [23] it was pointed out that the method presented is only applicable to a limited geometric range.

Comparison was made between the simplified approach of [24], a full transient finite element model and experiments, [27]. For two values of the fall height of the impactor and a fixed sized glass pane, the simplified approach yielded satisfying results. The verification was made with a glass pane width less than two meters.

In this work, a reduced finite element model for strength design of glass structures subjected to dynamic impact load was developed. The Rayleigh-Ritz procedure using generalized coordinates and Ritz vectors is used for modeling of the glass pane. Comparing the use of Ritz vectors to the use of for instance eigenvectors, there is the possibility to construct the Ritz vectors to account for the physical behavior of the system such as the position of the load. Thus, in general fewer vectors are needed for an accurate representation of the system. The model is similar to the model of for instance [22], but includes an additional Ritz vector in the representation of the glass pane. This means that the model error is reduced, especially for off-center loading. The second Ritz vector is constructed based on knowledge about the physical behavior of the system. This second vector is a key feature of the present model. The developed model is intended for integration into a glass design computer program, [17], which allows fast and accurate strength design of glass structures. The entire procedure is finite element based and in the computer program the user only has to give basic text based input data. The procedure can be used for different fall heights of the impactor, various support conditions and locations of the impact. Below, the accuracy of the model reduction technique is demonstrated for the case of centrically applied impact and four-sided supported glass. Following this, it is demonstrated that the model is applicable to eccentrically applied load cases. A final example illustrates the applicability of the model to a standard laminated glass balustrade with clamp fixings. It is a subject for future work to evaluate the model performance for the types of support conditions not covered in this study and to make further experimental verification.

2. A pendulum impact test for classifying glass for impact strength

The experimental test method used for classifying glass for impact strength, [26], is shown in Fig. 1(a). The arrangement consists of a glass pane held within a steel frame and an impactor consisting of a weight encased in two tires. Between the steel frame and the glass there are rubber strips. At testing, the tire is released into a pendulum motion subsequently impacting the glass pane. The dimensions of the frame are standardized to $1.95 \times 0.887\ \mathrm{m}^2$ and the weight of the impactor is 50 kg according to the standard. The test is considered as a soft impact with a long pulse time, Fig. 1(b).

3. A reduced model for glass panes subjected to dynamic impact loads

In the reduced dynamic modeling of the glass pane, including the supports, the Rayleigh-Ritz method, [7], was adopted. Consequently, the displacements, $\mathbf{u}(t)$, were expressed as a linear combination of shape (Ritz) vectors ψ_j:

$$\mathbf{u}(t) = \sum_{j=1}^{J} z_j(t)\psi_j = \Psi\mathbf{z}(t), \tag{1}$$

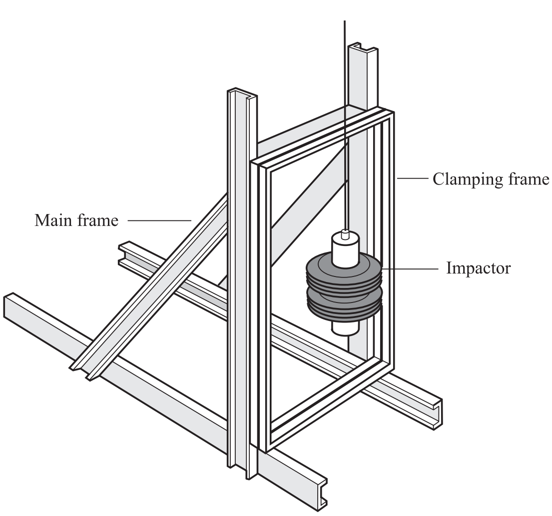
where $z_j(t)$ are the generalized coordinates.

The glass pane together with the supports can be represented as a multi-degree-of-freedom system with one degree of freedom for each Ritz vector. The equations of motion for an undamped multi-degree-of-freedom system in free vibration are given as, [7],

$$\mathbf{m}\ddot{\mathbf{u}} + \mathbf{k}\mathbf{u} = \mathbf{0}, \tag{2}$$

where $\mathbf{m}$ is the mass matrix and $\mathbf{k}$ is the stiffness matrix. Substituting (1) into (2) yields

$$\mathbf{m}\Psi\ddot{\mathbf{z}} + \mathbf{k}\Psi\mathbf{z} = \mathbf{0}. \tag{3}$$



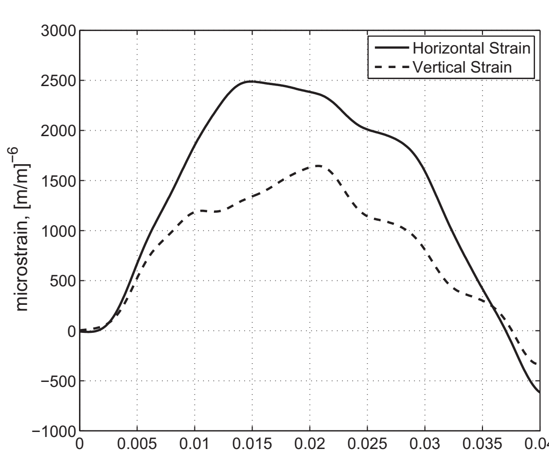


Fig. 1. Test arrangement for pendulum impact test (a) and example of result (b), [20].