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# Analysis of cylindrical sandwich structures with weak orthotropic core under patch loading

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ABSTRACT

An analysis of cylindrical sandwich structures with weak orthotropic core subjected to patch loading is presented. A high order theory model combined with a novel formulation technique is used to predict the static response of the structure. The face-sheets are considered as thin shells that follow the first order shear deformation theory, whereas the core is considered as a linear elastic medium. The effects of core elastic and shear stiffness, curvature to length ratio, and stacking sequence and orientation on the core's stresses and displacements are presented. The case of a moving load is explored and performance charts are generated to design and optimize the structure in response to the patch loads. In particular, considering an orthotropic core with quasi-isotropic lay-up in the face-sheets, limit the variation of the transverse core shear stress and provide an efficient structural design configuration.



## 1. Introduction

Modern engineering structures dictate the demand for light-weight yet stiff structural components, especially in the aerospace industry, military and civil infrastructure applications. Advanced composite materials have proven to be good candidates for providing the necessary performance requirements in terms of strength, stiffness, and weight. Sandwich composites emerged from within the realm of classical composite materials. Sandwich construction consists of top and bottom stiff skins or face-sheets separated by a relatively thick and lightweight core. The contribution of the core thickness and rigidity, as well as the skin–core interaction has been the subject for research ever since sandwich composites became of interest in structural applications.

Several theories have been developed to study the static and dynamic response of sandwich structures. The earliest theories were based on the classical approach in which the core was assumed to have a constant thickness before and after deformation. Such theories are well presented and reviewed in references such as Allen [1] and Zenkert [2]. While these theories do present a general idea of the global deflection, they fail to accurately capture the localized effects that arise due to the use of weak cores (Fig. 1).

Frostig [3] have developed the High-Order sandwich Theory (HOT), in which the core's height restriction was removed, allowing the core to extend or compress during deflection (Fig. 1). In the HOT, the face-sheets are modeled as separate beams, plates, or shells coupled by the core, which is considered as a linear elastic medium exhibiting only shear and vertical resistance, while the longitudinal stresses are ignored. The HOT proved to predict the behavior of a sandwich structure more accurately than its classical counterparts, mainly because the HOT does not place any assumptions on the core and thus allows the normal stresses to vary over the core's thickness.

Modern applications for sandwich composites are no longer restricted to flat panels, but have also extended to curved shapes, such as wind blades and fuselages on aerospace structures. Consequently, the HOT had to be extended to account for the effects of such structure's curvature.

Frostig [4] studied the bending of curved sandwich panels that incorporate a transversely flexible core. Later on, Frostig and Thomsen [5] used the HOT to study the bending behavior of a unidirectional generally curved soft sandwich panel. Afshin et al. [6] developed an analytical formulation for the HOT on cylindrical sandwich panels and investigated the applicability of this theory with comparison to Reddy's full layer-wise theory [7] as well as results from a finite element model.

Certain sandwich structures are subjected to concentrated loads commonly represented by patch loads or distributed loads. These loads are applied on specific bounded areas of the structure, such as lifting points or vertical chord members framing into an arched bridge span. Whitney [8] considered these cases for a flat panel. This paper attempts to study the behavior of cylindrical sandwich

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**Nomenclature**

| | | | |
|---|---|---|---|
| $A$ | in-plane stiffness matrix | $Q$ | shear resultant forces |
| $B$ | coupling stiffness matrix | $R_i$ | radius of curvature |
| $D$ | bending stiffness matrix | $t, b, c$ | in subscripts or superscripts refer to top, bottom, and core respectively |
| $E$ | modulus of elasticity | $\gamma_{x\theta}, \gamma_{xz}, \gamma_{z\theta}$ | shear strains |
| $G$ | shear modulus | $\varepsilon_{xx}, \varepsilon_{\theta\theta}, \varepsilon_{x\theta}$ | normal strains |
| HOT | High Order Theory | $\varkappa$ | strains related to shell rotation |
| $k$ | shear correction factor | $\sigma_{xx}, \sigma_{\theta\theta}, \sigma_{x\theta}$ | normal stresses |
| $M$ | bending resultant moments | $\tau_{x\theta}, \tau_{xz}, \tau_{z\theta}$ | shear stresses |
| $N$ | axial resultant forces | | |

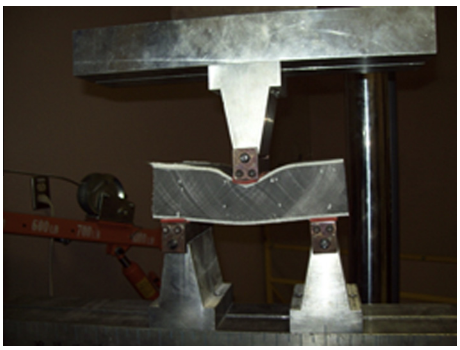

**Fig. 1.** Local compression of a sandwich beam with weak core in three-point bending test.

composites subjected to patch loads using the HOT. This study will extend the theory developed by Afshin et al. [6] to include the effects of the orthotropic core and uses Fourier's formulation to simulate the patch loads. The effects of varying the patch's load locations, the core stiffness, and the ply stacking sequence and orientation in the face-sheets will be investigated.

## 2. A cylindrical sandwich shell theory with orthotropic core

In this study, a cylindrical sandwich composite is considered. The global coordinate system of the structure, as well as the local coordinate system for each part of the sandwich are illustrated in Fig. 2.

The local coordinate systems, $z_t, z_c, z_b$ in each part of the sandwich composite are related to the global coordinate system by the relation: $z_i = r - R_i$, with $i = t, c, b$.

### 2.1. Laminate composite constitutive equations

The structure consists of composite face-sheets; therefore, a basic understanding of the constitutive equations of the composite laminate is required.

It is important to note that although the strains within the lamina's plies are continuous, the stress does not necessarily need to be continuous. Therefore, in order to obtain the force resultant in the lamina, a ply-by-ply through the thickness integration is required. The stress notations are referred in Fig. 3.

Mathematically, the force resultants, as mentioned in several textbooks such as Reddy [7], are expressed as:

$$\begin{Bmatrix} N_{xx} \\ N_{\theta\theta} \\ N_{x\theta} \\ M_{xx} \\ M_{\theta\theta} \\ M_{x\theta} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{26} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{0xx} \\ \varepsilon_{0\theta\theta} \\ \varepsilon_{0x\theta} \\ \varkappa_{xx} \\ \varkappa_{\theta\theta} \\ \varkappa_{x\theta} \end{Bmatrix} \tag{1}$$

Also,

$$\begin{Bmatrix} Q_{\theta z} \\ Q_{xz} \end{Bmatrix} = K \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{\theta z} \\ \gamma_{xz} \end{Bmatrix} \tag{2}$$

where,

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{N} \int_{z_k}^{z_k+1} Q_{ij}(1, z, z^2)\, dz \tag{3}$$

Here, $K$ is the shear correction factor which is used in order to allow the transverse shear stress's strain energy to be equal to that calculated from the 3-D elasticity theory, $\varepsilon_0$'s are the strains at the mid-plane, and $\gamma$'s are the shear strains.

### 2.2. High-order theory formulation

The sandwich composite is considered as thin-skinned composite. Therefore, these skins can be considered as thin shells, and the first-order shear deformation theory can be used to model their static behavior. The core on the other hand, is considered as a 3-dimensional orthotropic elastic medium. No assumptions are made on the deformation field within the core, which is the basis of the high-order theory; however, the in-plane stresses are neglected and only the transverse normal and shear stresses are taken into account. The next sections will closely follow the outline of the theory provided by Afshin et al. [6], and only the main differences on the major points of interest will be mentioned in this paper.

Following the first order shear deformation theory, the following assumptions for the face-sheets and the core are required:

1. The faces are assumed to be perfectly bonded to the core. Therefore, no delamination or skin slip exists at the core–skin interface.
2. Both the core and skins are made of linear elastic materials that exhibit small deformations; therefore, the kinematic relations in those materials are linear.
3. The face thicknesses, which are not required to be equal, are small with respect to the sandwich's radii of curvature. This assumption is also a requirement, to ensure that the principle of thin shell theory is valid, and that the relation $1 + z/R \approx 1$ would be a reasonable approximation.

Consequently, the $x$, $\theta$, and $z$ deformations for the top and bottom faces can be respectively written as:

$$\begin{aligned} u_x^j(x,\theta,z) &= u_0^j(x,\theta) + z\varphi_1^j(x,\theta) \\ v_\theta^j(x,\theta,z) &= v_0^j(x,\theta) + z\varphi_2^j(x,\theta) \\ w_z^j(x,\theta,z) &= w_0^j(x,\theta) \end{aligned} \tag{4}$$

where $j = t, b$.