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Nonlinear bending of nanotube-reinforced composite cylindrical panels resting on elastic foundations in thermal environments

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ABSTRACT

Nonlinear bending analysis is presented for nanocomposite cylindrical panels subjected to a transverse uniform or sinusoidal load resting on elastic foundations in thermal environments. Carbon nanotubes are used to reinforce the cylindrical panels in two distinguished patterns, namely, uniformly distributed (UD) and functionally graded (FG) reinforcements. The material properties of CNTRCs are assumed to be temperature-dependent and are estimated by a micromechanical model. The governing equations of the panel are derived based on a higher-order shear deformation theory with a von Kármán-type of kinematic nonlinearity and are solved by a two-step perturbation technique. The nonlinear bending behaviors of the CNTRC panels with different CNT volume fraction distributions, foundation stiffnesses, temperature rise, and the character of in-plane boundary conditions are studied in details.

1. Introduction

Carbon nanotube-reinforced composites (CNTRCs) are a new generation of advanced composite materials that have attracted the great attention of material scientists and engineers. Owing to their remarkable mechanical, electrical and thermal properties, these new nanocomposite materials can be used to make key components in Micro-Electro-Mechanical Systems (MEMS) and Nano-Electro-Mechanical Systems (NEMS) [1,2]. When using carbon nanotubes to replace traditional carbon fibers as the reinforcements in the composites, a small weight percentage of the carbon nanotubes (2–5%) can significantly increase the stiffness and strength of the composites [3–6]. On the other hand, adding more CNTs to the composites can actually lead to the deterioration of their mechanical properties [7]. In order to effectively make use of the low weight percentage of CNTs in the composites, Shen [8] introduced the functionally graded material concept into the CNTRC composites in his study on the nonlinear bending behavior of CNTRC plates. He used a linear distribution pattern of CNTs along the thickness direction of the plates and found that the functionally graded CNT reinforcements can significantly change the bending behavior of the CNTRC plates. The concept of functionally graded nanocomposites has been realized in a recent experimental study [9] where a functionally graded CNT reinforced aluminum composite was fabricated using a powder metallurgy route. These pioneer works have stimulated extensive studies on the bending, buckling and vibration responses of CNTRC structures in recent years [10–23].

Since this area is relatively new, published literature on the bending, buckling and vibration responses of CNTRC shell panels is limited and most of them are focused on the cases of linear problems. Aragh et al. [24] investigated the linear free vibration of functionally graded CNTRC cylindrical panels based on the Eshelby–Mori–Tanaka approach. It is worth noting that the Eshelby–Mori–Tanaka model is applicable to agglomerated carbon nanotubes, but is not applicable to aligned carbon nanotubes. Using the Voigt model as the rule of mixture for determining the CNTRC composite material properties, Jam et al. [25] and Yas et al. [26] carried out studies on the linear free vibration of functionally graded CNTRC cylindrical panels using 3-D elasticity theory. On the other hand, Lei et al. [27] studied the large deflection of functionally graded CNTRC plates using the element-free kp-Ritz method. In their analysis, the material properties of functionally graded CNTRCs are assumed to be graded in the thickness direction and are estimated through an equivalent continuum model based on the Eshelby–Mori–Tanaka approach. Liew et al. [28] performed a postbuckling analysis of functionally graded CNTRC cylindrical panels. In their analysis, formulations are based on the first order shear deformation shell theory and are solved by using the arc-length method combined with the modified Newton–Raphson method. In all the aforementioned studies

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[24–28], however, the effective material properties of CNTRCs are assumed to be independent of temperature. Recently, Shen and Xiang [29,30] investigated the large amplitude vibration and compressive postbuckling of functionally graded CNTRC cylindrical panels resting on elastic foundations in thermal environments. In their analysis, formulations are based on the higher order shear deformation shell theory and the material properties of CNTRCs are assumed to be temperature-dependent.

In the present work, we focus our attention on the nonlinear bending of CNTRC cylindrical panels resting on elastic foundations in thermal environmental conditions. Two kinds of CNTRC cylindrical panels, namely, uniformly distributed (UD) and functionally graded (FG) reinforcements, are considered. The panel is exposed to elevated temperature and is subjected to a transverse uniform or sinusoidal load. The material properties of FG-CNTRCs are assumed to be graded in the thickness direction, and are estimated through a micromechanical model. The formulations are based on a higher order shear deformation shell theory and general von Kármán-type equation that includes the panel–foundation interaction and thermal effects. A two-step perturbation approach is employed to determine the load–deflection and load–bending moment curves. The numerical illustrations show the nonlinear bending response of CNTRC cylindrical panels resting on elastic foundations under different sets of loading and temperature conditions.

2. Theoretical development

Consider a CNTRC cylindrical panel resting on an elastic foundation. The panel is referenced to a coordinate system (X,Y,Z) in which X and Y are in the axial and circumferential directions of the panel and Z is in the direction of the inward normal to the middle surface, and the corresponding displacements are designated by $\overline{U}$, $\overline{V}$, and $\overline{W}$. $\overline{\Psi}_x$ and $\overline{\Psi}_y$ are the rotations of the normals to the middle surface with respect to the Y- and X-axes, respectively. The origin of the coordinate system is located at the corner of the panel in the middle plane. As shown in Fig. 1, R is the radius of curvature, h the panel thickness, a the length in the X direction, and b the length in the Y direction, respectively. The panel is exposed to elevated temperature and is subjected to a transverse uniform load $q = q_0$ or a sinusoidal load $q = q_0 \sin(\pi X/a) \sin(\pi Y/b)$. As is customary [31–33], the foundation is assumed to be a compliant foundation, which means that no part of the panel lifts off the foundation in the large deflection region. The load–displacement relationship of the foundation is assumed to be $p_0 = \overline{K}_1\overline{W} - \overline{K}_2\nabla^2\overline{W}$, where p_0 is the force per unit area, $\overline{K}_1$ is the Winkler foundation stiffness and $\overline{K}_2$ is the shearing layer stiffness of the foundation, and ∇^2 is the Laplace operator in X and Y. Let $\overline{F}(X,Y)$ be the stress function for the stress resultants defined by $\overline{N}_x = \overline{F}_{,YY}$, $\overline{N}_y = \overline{F}_{,XX}$ and $\overline{N}_{xy} = -\overline{F}_{,XY}$, where a comma denotes partial differentiation with respect to the corresponding coordinates.

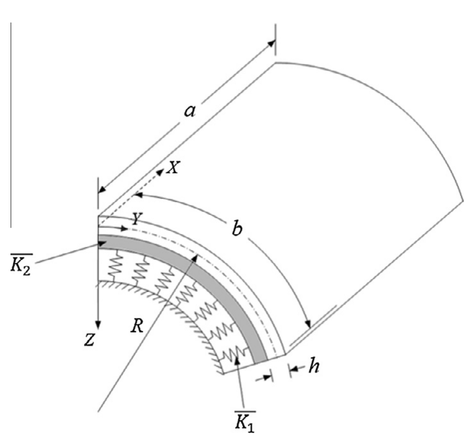

Fig. 1. Geometry and coordinate system of cylindrical panel on a Pasternak elastic foundation.

Reddy and Liu [34] developed a simple higher order shear deformation shell theory based on the Sanders shell theory. This theory assumes that the transverse shear strains are parabolically distributed across the shell thickness. The advantages of this theory over the first order shear deformation theory are that the number of independent unknowns ($\overline{U}, \overline{V}, \overline{W}, \overline{\Psi}_x$ and $\overline{\Psi}_y$) is the same as in the first order shear deformation theory, but no shear correction factors are required. Based on Reddy's higher order shear deformation shell theory with a von Kármán-type of kinematic nonlinearity and including panel–foundation interaction and thermal effects, the governing equations for an FG-CNTRC cylindrical panel can be derived in terms of a stress function $\overline{F}$, two rotations $\overline{\Psi}_x$ and $\overline{\Psi}_y$, and a transverse displacement $\overline{W}$. They are

$$\widetilde{L}_{11}(\overline{W}) - \widetilde{L}_{12}(\overline{\Psi}_x) - \widetilde{L}_{13}(\overline{\Psi}_y) + \widetilde{L}_{14}(\overline{F}) - \widetilde{L}_{15}(\overline{N}^T) - \widetilde{L}_{16}(\overline{M}^T) - \frac{1}{R}\overline{F}_{,XX} + \overline{K}_1\overline{W} - \overline{K}_2\nabla^2\overline{W} = \widetilde{L}(\overline{W},\overline{F}) + q \tag{1}$$

$$\widetilde{L}_{21}(\overline{F}) + \widetilde{L}_{22}(\overline{\Psi}_x) + \widetilde{L}_{23}(\overline{\Psi}_y) - \widetilde{L}_{24}(\overline{W}) - \widetilde{L}_{25}(\overline{N}^T) + \frac{1}{R}\overline{W}_{,XX} = -\frac{1}{2}\widetilde{L}(\overline{W},\overline{W}) \tag{2}$$

$$\widetilde{L}_{31}(\overline{W}) + \widetilde{L}_{32}(\overline{\Psi}_x) - \widetilde{L}_{33}(\overline{\Psi}_y) + \widetilde{L}_{34}(\overline{F}) - \widetilde{L}_{35}(\overline{N}^T) - \widetilde{L}_{36}(\overline{S}^T) = 0 \tag{3}$$

$$\widetilde{L}_{41}(\overline{W}) - \widetilde{L}_{42}(\overline{\Psi}_x) + \widetilde{L}_{43}(\overline{\Psi}_y) + \widetilde{L}_{44}(\overline{F}) - \widetilde{L}_{45}(\overline{N}^T) - \widetilde{L}_{46}(\overline{S}^T) = 0 \tag{4}$$

in which

$$\widetilde{L}() = \frac{\partial^2}{\partial X^2}\frac{\partial^2}{\partial Y^2} - 2\frac{\partial^2}{\partial X\partial Y}\frac{\partial^2}{\partial X\partial Y} + \frac{\partial^2}{\partial Y^2}\frac{\partial^2}{\partial X^2} \tag{5}$$

and the other linear operators $\widetilde{L}_{ij}()$ are defined as in [35]. Note that the geometric nonlinearity in the von Kármán sense is given in terms of $\widetilde{L}()$ in Eqs. (1) and (2). It is worthy to note that the governing Eqs. (1)–(4) for an FG-CNTRC cylindrical panel are identical in form to those of unsymmetric cross-ply laminated panels.

In the above equations, $\overline{N}^T$, $\overline{M}^T$, $\overline{S}^T$ and $\overline{P}^T$ are the forces, moments and higher order moments caused by the elevated temperature, and are defined by

$$\begin{bmatrix} \overline{N}_x^T & \overline{M}_x^T & \overline{P}_x^T \\ \overline{N}_y^T & \overline{M}_y^T & \overline{P}_y^T \\ \overline{N}_{xy}^T & \overline{M}_{xy}^T & \overline{P}_{xy}^T \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} A_x \\ A_y \\ A_{xy} \end{bmatrix} (1, Z, Z^3)\Delta T\, dZ \tag{6a}$$

$$\begin{bmatrix} \overline{S}_x^T \\ \overline{S}_y^T \\ \overline{S}_{xy}^T \end{bmatrix} = \begin{bmatrix} \overline{M}_x^T \\ \overline{M}_y^T \\ \overline{M}_{xy}^T \end{bmatrix} - \frac{4}{3h^2}\begin{bmatrix} \overline{P}_x^T \\ \overline{P}_y^T \\ \overline{P}_{xy}^T \end{bmatrix} \tag{6b}$$

where $\Delta T = T - T_0$ is the temperature rise from some reference temperature T_0 at which there are no thermal strains, and

$$\begin{bmatrix} A_x \\ A_y \\ A_{xy} \end{bmatrix} = -\begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \alpha_{11} \\ \alpha_{22} \end{bmatrix} \tag{7}$$

where α_{11} and α_{22} are the thermal expansion coefficients measured in the longitudinal and transverse directions, and $\overline{Q}_{ij}$ are the transformed elastic constants with details being given in [34]. Note that for an FG-CNTRC layer, $\overline{Q}_{ij} = Q_{ij}$ in which

$$Q_{11} = \frac{E_{11}}{1-\nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_{22}}{1-\nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}},$$
$$Q_{16} = Q_{26} = 0, \; Q_{44} = G_{23}, \; Q_{55} = G_{13}, \; Q_{66} = G_{12}, \tag{8}$$