



A higher order steel–concrete composite beam model



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ABSTRACT

This paper presents a model for the analysis of composite beams in which the constrained kinematics encompasses the overall shear deformability, warping of the slab cross section and of the steel beam and partial shear interaction between slab and girder. The warping functions are obtained by considering the problem of unrestrained thin-walled members subjected to self-equilibrated elementary load schemes. The governing equations are derived, according to the stiffness method, both in the weak and strong forms starting from the Virtual Work Theorem which makes it possible to consistently obtain the resultants of stresses, the applied forces and the inertia involved in the problem. The analytical solution is obtained from the governing differential equations and the relevant boundary conditions exploiting exponential matrices. Some simple applications show the capability of the model to accurately describe both the global behaviour of composite beams, in terms of displacements and stress resultants, and local effects, in terms of normal stresses. The comparisons with results obtained with a refined shell finite element model are very satisfactory.

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1. Introduction

Steel–concrete composite beams are widely employed in many civil engineering applications, such as decks in buildings, steel framed structures and bridges. When a concrete slab is coupled to a steel beam the advantages of both materials are combined and the mechanical behaviour of the composite member is optimised. In the case of bridge decks, the concrete slab is very large and thin while the steel member is formed by two or more I-beams or box girders with high and very thin webs. The composite action is assured by the connection at the slab–beam interface which has to transfer the shear force between the two components. This is usually constituted by stud connectors, welded at the top flange of the steel girder, characterised by certain shear deformability. This system does not completely prevent relative longitudinal displacements (slip) between the two components while practically avoid their separation (uplift).

An accurate evaluation of the beam deflection, forces on the connection, and stresses on the slab and steel beam, which are required for many verifications (e.g., serviceability limit states, fatigue assessment, some ultimate limit states), cannot be obtained with traditional beam theories based on the assumption of the

preservation of the plane cross section. As an alternative to Finite Element (FE) models based on planar or solid elements, the analysis of such structures may be performed with higher-order beam models able to capture the effects of the deformable connection, the shear deformability of the overall cross section and the shear-lag produced by the warping of the slab and steel beam cross-sections.

The theoretical literature on steel–concrete composite beams is impressive, dealing with different analytical approaches, solution formulations, types of analyses and applications. A real state of art is beyond the scope of this paper and a short literature review is reported focusing on the theoretical modelling. Starting from the seminal paper by Newmark et al. [1], in which the composite beam is modelled by coupling two Euler–Bernoulli members by means of a deformable shear connection distributed along their interface, many beam theories have been developed to account for different kinematic aspects. Models accounting for the uplift were developed by several authors who demonstrated that the global response of a composite beam is rather unaffected by the uplift [2] whereas local uplift effects induced by concentrated loads may be important as shown by Gara et al. [3] on the basis of their displacement based novel formulation. With regard to the shear-lag effect many models were developed, some accounting only for the warping of the slab and others also for that of the steel flanges; among the first, Dezi et al. [4–6] described the slab warping by means of shape functions derived for composite decks

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with twin I-girders or single box-girder according to the Reissner approach [7] which is still widely adopted in several recent higher order models for homogeneous beams [8–10]. The above mentioned models are appropriate to investigate local effects due to the slab–beam connection and to catch the stress concentration on the slab or steel flanges due to shear-lag but cannot adequately describe the overall deformability of the composite beams which is strongly influenced by shear deformability, especially in the case of continuous viaducts with reduced span-to-depth ratio.

By assuming that shear forces on the concrete slab and the steel beam are proportional to their stiffness, Xu and Wu [11] developed a model where slab and beam behave as two Timoshenko elements in which shear deformations and flexural rotations are constrained to be equal [12]. Ranzi and Zona [13] considered an Euler–Bernoulli beam for the concrete slab and a Timoshenko beam for the steel member since the shear deformation of the slab is expected to be negligible due to its slenderness. These models introduce constraints that are acceptable in most of the civil engineering applications but that may lead to unrealistic results in some cases; for instance in composite elements made of materials with high stiffness contrast, the cross section rotations of the two elements may be significantly different. Several authors developed models accounting for shear deformability of both components coupling two Timoshenko beams by means of a deformable shear connection without introducing any constraint on the cross section rotations [14,15]. Despite these models can predict the global behaviour with acceptable accuracy, they cannot capture the non-uniform stress distribution (shear-lag effect) as the cross section warping is neglected.

A model, accounting for both shear deformation and cross section warping of the two components, has been recently developed by Chakrabarti et al. [16] making use of the higher-order shear deformation theory proposed by Reddy [17] for laminated composite beams. The higher-order terms introduced in the displacement field are represented by two a-priori fixed functions (second and third order polynomials) which can only describe the cross section warping within the depth; this cannot capture the typical shear-lag due to the in-plane behaviour of slab and flanges which can be significant for beams composed by a wide slab connected to box or I-shaped girders along longitudinal lines, such as in typical composite bridge decks. Although more accurate models have been recently developed for multilayered beams by Vo and Thai [18] and by Silvestre and Camotin [19], higher-order models for steel–concrete beams with deformable interlayer connection are not available in the literature.

Besides the theoretical approaches, the literature is also rich of numerical procedures to implement the models previously described. Different types of analyses, such as linear elastic and viscoelastic [20], non-linear [21–23], dynamic and buckling [11] have been developed. Closed form solutions are available only for some cases [24–27] while, in general, a number of numerical solutions have been proposed based on different methods such as the direct stiffness approach (stiffness matrix) [28,29], the finite element method [15,30–34] and the finite difference formulation [4,5].

In this paper a higher-order formulation for the analysis of composite beams with partial shear interaction between the slab and the girder is proposed. The model, which can be considered a generalisation of the Newmark model [1], takes into account both the overall shear deformability of the components [12] and the warping of the slab cross section and the steel beam [7]. Two different rotations are considered to avoid introducing a constraint between the two components. As for the warping of the cross section, special shape functions are obtained by considering the problem of unrestrained thin-walled members subjected to self-equilibrated elementary load schemes. In particular, three different functions are introduced: one for the membrane action of the concrete slab,

due to the longitudinal flow at the shear connection, and other two functions for the steel element, to catch separately the warping due to the shear force component resisted by the steel beam and to the shear flow at the slab–beam interface [6,35,36]. The governing equations are derived, according to the stiffness method, both in the weak and strong forms starting from the Virtual Work Theorem which makes it possible to consistently obtain the resultants of stresses, the applied forces and the inertia involved in the problem. The rigorous solution of the system of differential equations governing the problem is obtained exploiting exponential matrices. This method, furnishing the exact formulation of the solution without requiring any discretisation of the beam, is used to determine the solution of the proposed applications according to the stiffness method deriving the stiffness matrix and the reactions of fixed-end beams. The model is validated with reference to a realistic two-span bridge by comparing solutions with results provided by refined shell finite element models.

2. Analytical model

2.1. Kinematics

A prismatic composite girder with symmetric cross section, obtained by connecting a steel beam with a concrete upper slab, is considered (Fig. 1). The external loads are assumed to be applied in compliance with the geometric symmetry in order to avoid torsion and transverse displacements. An orthonormal global reference frame $\{0; X, Y, Z\}$ is chosen so that the beam axis is parallel to the direction Z and the symmetry plane of the beam lies on the co-ordinate plane YZ . The slab is a prismatic element with a rectangular cross section of width $2B$ and thickness t_c and is connected to the steel girder along two lines at coordinates (\tilde{x}, \tilde{y}) and $(-\tilde{x}, \tilde{y})$; it is considered to behave like a solid element under bending and like a thin-walled element under membrane actions. The steel component is considered to be a thin-walled element because of its three orders of dimensions (length, mean cross section dimension and wall thickness); more specifically, it is constituted by n plane walls, with constant thickness t_i , for which the local abscissa $\xi_i \in [0, L_i]$, running along the section contour, is introduced (Fig. 1(b)). Despite local abscissas are defined for every wall, if not differently specified, index i will be omitted hereafter for ease of notation.

By assuming that the composite cross section is transversally rigid, and that the interface shear connection is stiff against uplift, the vertical admissible displacement of each point is equal to the beam axis deflection v_0

$$v(x, y, z) = v_0(z) \quad (1)$$

With reference to the concrete slab, the following admissible longitudinal displacements are considered:

$$w_c(x, y, z) = \mathbf{a}_c(x, y) \cdot \mathbf{w}_c(z) \quad (2)$$

where

$$\mathbf{a}_c^T(x, y) = [1 \quad y \quad \psi_c(x)] \quad (3)$$

is a geometric vector in which ψ_c is a known warping function, and

$$\mathbf{w}_c^T(z) = [w_{c0}(z) \quad \phi_c(z) \quad f_c(z)] \quad (4)$$

is the vector that groups together the slab generalised displacements, namely the longitudinal displacement w_{c0} measured at the Z axis, the rotation ϕ_c with respect to the X axis and the warping intensity f_c . For the steel beam, the longitudinal displacements are referred to the wall contour and are defined as

$$w_s(\xi_i, z) = \mathbf{a}_s(\xi_i) \cdot \mathbf{w}_s(z) \quad i = 1, \dots, n \quad (5)$$

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