Engineering Structures 87 (2015) 1–7

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Transient dynamic analysis of shear deformable shallow shells using the boundary element method

J. Useche [a,*], E.L. Albuquerque [b]

[a] Department of Mechanical Engineering, Technological University of Bolivar, Cartagena, Colombia
[b] Department of Mechanical Engineering, University of Brasilia, Brasilia, Brazil

ARTICLE INFO

Article history:
Received 23 July 2014
Revised 4 November 2014
Accepted 3 January 2015
Available online 28 January 2015

Keywords:
Boundary element method
Shear deformable shallow shells
Transient dynamic analysis
Dual Reciprocity Boundary Element Method
Houbolt method

ABSTRACT

Dynamic plate bending problems appear on civil, mechanical, aerospatial and naval applications. The complexity involved in the dynamic response of plates brings many challenges from a mathematical standpoint. In this work, the transient dynamic analysis of elastic shallow shells under uniformly distributed pressure loads, using a dual reciprocity boundary element formulation, is presented. A boundary element formulation based on a direct time-domain formulation using elastostatic fundamental solutions was used. Effects of shear deformation and rotatory inertia were included in the formulation. Shallow shells are modeled coupling boundary element formulation of shear deformable plate and two-dimensional plane stress elasticity. Domain integrals related to inertial terms were treated using the Dual Reciprocity Boundary Element Method. Numerical examples are presented to demonstrate the efficiency and accuracy of the proposed formulation.

1. Introduction

Complex spatial structures as found in aerospatial, naval and automotive applications, are made of assembled plates and shells. In general, these structures have constructive details such as holes, folds and joint stiffeners and difficult mesh generation when finite element method based are models are used. Moreover, mesh refinement is required in the neighborhood of such details, arising the computational cost. For this reason, the use of the boundary element method represents and valid alternative to domain discretization methods for the analysis of these kind of structures [30]. Aerospatial, naval and automotive structures, among others, are required to support dynamic loads. The complexity involved in the dynamic response of shells is challenging and problematic from a mathematical standpoint. In general, numerical methods represent the only way to obtain approximate solutions for dynamic analysis. Dynamic analysis of shells using the Finite Element Method (FEM) is well established [32,1,10,16,14,17]. However, FEM requires refined meshes since the length of the elements should be proportional to the size of the wavelength. This means a high number of degrees of freedom, which requires a significant computational effort.

Within the last decade meshless methods for the dynamic analysis of plates and shells have been proposed in order to overcome problems related with domain discretization methods [3,7,31,9,2,29]. However, their most important drawback of meshless methods relies on their high computational costs and occasional instabilities that appear in certain meshfree methods.

Alternatively, the Boundary Element Method (BEM) has emerged as an accurate and efficient numerical method for shear deformable plate and shell static analysis [30,15,4,28,27]. BEM has become important in the structural analysis of complex geometries such as ships and aircraft, where the use of domain discretization methods such as FEM have a high computational cost. Unlike to domain discretization techniques, the process of discretization in the Boundary Element Method (BEM) takes place only on the boundary, so that the system of equations is much smaller and less time is required to identify a solution to a problem. Moreover, in domain discretization techniques, finer meshes must be used in high stress concentration regions. In BEM, no interpolation are needed for stress calculation and the quality of the solution is solely dependent on the quality of boundary solution for displacement. As such, a decrease in unwanted information is present than in other numerical techniques provided that interior domain variables appear where only they are required. However, time consumed by FEM and BEM for a specific analysis is strongly depend on how the numerical method is computationally implemented. As we have not worried about the efficiency of the

* Corresponding author. Tel.: +57 5 6535216; fax: +57 5 6619240.
E-mail addresses: juseche@unitecnologica.edu.co (J. Useche), eder@unb.br (E.L. Albuquerque).

http://dx.doi.org/10.1016/j.engstruct.2015.01.006
0141-0296/

code at this stage, a comparison of time consumed by FEM and BEM represent a difficult task.

Dynamic BEM analysis of shear deformable plates using elastodynamic fundamental solutions, Laplace or Fourier transformations of these fundamental solutions were used in [8,6,13,24–26]. In [20–22,12] a time-domain direct BEM formulations based on elastostatic fundamental solutions for dynamic analysis of shear deformable plates are presented. To date, very few publications demonstrate the dynamic analysis of plate or shell structures using the boundary element method analysis. A boundary element method formulation for dynamic analysis of shells represents a contribution to the structural analysis of complex structures.

This work presents the dynamic analysis of shear deformable elastic shallow shells under uniformly distributed pressure loads, using a boundary element formulation. This formulation is based on direct time integration and elastostatic fundamental solutions. Effects of shear deformation and rotatory inertia are included in the formulation. Shells were modeled by coupling the boundary element formulation for shear deformable plates based on the Reissner plate theory and two-dimensional plane stress elasticity, as presented in [4,28]. The Dual Reciprocity Boundary Element Method for the treatment of domain integrals involving inertial mass, was used. In order to obtain the time response, the Houbolt integration scheme was used. Numerical examples are presented and results were compared with those obtained using finite element models.

2. Shallow shell dynamic equations

Consider a shallow shell of uniform thickness h, mass density ρ and curvature radius $R_{\alpha\beta}$, occupying an area Ω, in the x_1x_2 plane, bounded by a contour $\Gamma = \Gamma_w \bigcup \Gamma_q$ with $\Gamma = \Gamma_w \bigcap \Gamma_q \equiv 0$, as presented in Fig. 1. The dynamic bending response for the shallow shell was modeled coupling the classical Reissner plate theory and the two-dimensional plane stress elasticity as presented in [27].

Equations of motion for an infinitesimal plate element are given by [16]:

$$\mathcal{L}^b_{ik} w_k + q^*_i = \Lambda^b_{ik}\ddot{w}_k + \Lambda^{bm}_{i\alpha}\ddot{u}_\alpha \tag{1}$$

$$\mathcal{L}^m_{\alpha\beta} u_\beta = \Lambda^{bm}_{\alpha\beta}\ddot{w}_\beta + \Lambda^m_{\alpha\beta}\ddot{u}_\beta \tag{2}$$

Indicial notation is used throughout this work. Greek indices vary from 1 to 2 and Latin indices take values from 1 to 3. Einstein's summation convention is used unless otherwise indicated. In these equations, w_α represents rotations with respect to x_1 and x_2 axes, and w_3 represents transverse deflection; $\ddot{w}_\alpha$ denotes angular accelerations corresponding to x_1 and x_2 axes, respectively, $\ddot{w}_3$ represents the transverse linear acceleration; u_α and $\ddot{u}_\alpha$ represents membrane displacements and accelerations along x_α axis, respectively; Tensors Λ^b_{ij}, Λ^{bm}_{ij} and Λ^m_{ij} are defined as: $\Lambda^b_{\alpha\beta} = I_2\delta_{\alpha\beta}$ and $\Lambda^b_{33} = I_0$; $\Lambda^{bm}_{\alpha\beta} = I_1\delta_{\alpha\beta}$; $\Lambda^m_{\alpha\beta} = I_0\delta_{\alpha\beta}$ and $\Lambda^m_{3i} = \Lambda^m_{i3} = \Lambda^b_{3i} = \Lambda^b_{i3} = \Lambda^{bm}_{3i} = \Lambda^{bm}_{i3} = 0$; $\delta_{\alpha\beta}$ is the Kronecker's delta and I_i are the mass inertias [16]. In these equations $\mathcal{L}^b_{ik}$ and $\mathcal{L}^m_{ik}$ operators are given in [4] and q^*_i is the equivalent body force ($q^*_\alpha = 0$):

$$q^*_3 = q_3 - B(\kappa_{11} + \nu\kappa_{22})u_{\alpha,\alpha} - B(\kappa^2_{11} + \nu\kappa^2_{22} + 2\nu\kappa_{11}\kappa_{22})w_3 \tag{3}$$

where q_3 is a distributed transverse load and $\kappa_{\alpha\beta} = 1/R_{\alpha\beta}$ represents the curvature tensor of the shell surface.

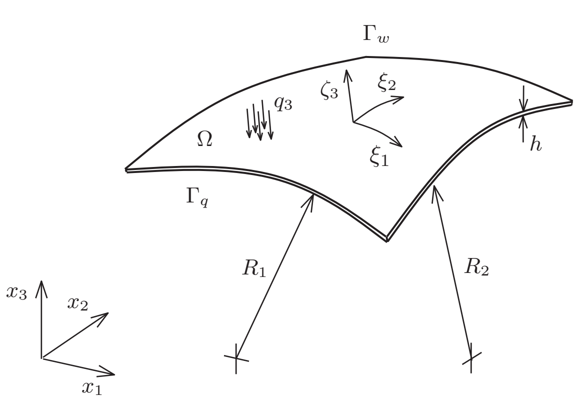

Fig. 1. Shallow shell geometry.

3. Boundary integral formulation for shallow shells

The derivation of the integral formulation for Eqs. (1) and (2) is based on application of the boundary element method to the Reissner plate theory as presented in [23], where the integral representations related to the governing equations for bending and transverse shear stress resultants are derived by using the weighted residual method, and making use of the Green's identity. Thus, by integration of Eq. (1), the following equations are obtained:

$$\begin{aligned} &c_{ij}w_j(\mathbf{x}') + \int_\Gamma P_{ij}(\mathbf{x}',\mathbf{x})w_j(\mathbf{x})d\Gamma \\ &= \int_\Gamma W_{ij}(\mathbf{x}',\mathbf{x})p_j(\mathbf{x})d\Gamma \\ &\quad - \int_\Gamma \kappa_{\alpha\beta}B\frac{1-\nu}{2}\left[u_\alpha(\mathbf{x})n_\beta + u_\beta(\mathbf{x})n_\alpha + \frac{2\nu}{1-\nu}u_\gamma(\mathbf{x})n_\gamma\delta_{\alpha\beta}\right]W_{i3}(\mathbf{x}',\mathbf{x})d\Gamma \\ &\quad + \int_\Omega \kappa_{\alpha\beta}B\frac{1-\nu}{2}\left[u_\alpha(\mathbf{X})W_{i3,\beta}(\mathbf{x}',\mathbf{X}) + u_\beta(\mathbf{x})W_{i3,\alpha}(\mathbf{x}',\mathbf{X})\right. \\ &\quad \left. + \frac{2\nu}{1-\nu}u_\gamma(\mathbf{x})W_{i3,\gamma}(\mathbf{x}',\mathbf{X})\delta_{\alpha\beta}\right]d\Omega \\ &\quad - \int_\Omega \kappa_{\alpha\beta}B[(1-\nu)\kappa_{\alpha\beta} + \nu\delta_{\alpha\beta}\kappa_{\gamma\gamma}]w_3(\mathbf{X})W_{i3}(\mathbf{x}',\mathbf{X})d\Omega \\ &\quad + \int_\Omega W_{i3}(\mathbf{x}',\mathbf{X})q_3(\mathbf{x})d\Omega + \int_\Omega W_{ij}(\mathbf{x}',\mathbf{x})\Lambda^b_{jk}\ddot{w}_k(\mathbf{X})d\Omega \\ &\quad + \int_\Omega W_{i\alpha}(\mathbf{x}',\mathbf{X})\Lambda^{bm}_{\alpha\beta}\ddot{u}_\beta(\mathbf{X})d\Omega \end{aligned} \tag{4}$$

Similarly, the derivation of the integral formulation for Eq. (2) is based on application of the boundary element method to the two-dimensional elasticity equations, as presented in [4]. Thus, by integration of Eq. (2) the following equations are obtained:

$$\begin{aligned} &c_{\theta\alpha}(\mathbf{x}')u_\alpha(\mathbf{x}') + \int_\Gamma T_{\theta\alpha}(\mathbf{x}',\mathbf{x})u_\alpha(\mathbf{x})d\Gamma \\ &\quad + \int_\Omega U_{\theta\alpha,\beta}(\mathbf{x}',\mathbf{X})B[\kappa_{\alpha\beta}(1-\nu) + \nu\delta_{\alpha\beta}\kappa_{\gamma\gamma}]w_3(\mathbf{X})d\Omega \\ &= \int_\Gamma U_{\theta\alpha}(\mathbf{x}',\mathbf{x})t_\alpha(\mathbf{x})d\Gamma + \int_\Omega U_{\theta\alpha}(\mathbf{x}',\mathbf{X})\Lambda^m_{\theta\alpha}\ddot{u}_\alpha(\mathbf{X})d\Omega \\ &\quad + \int_\Omega U_{\theta\alpha}(\mathbf{x}',\mathbf{X})\Lambda^{bm}_{\theta\alpha}\ddot{w}_\alpha(\mathbf{X})d\Omega \end{aligned} \tag{5}$$

In these equations, $\mathbf{x}'$ and $\mathbf{x}$ represent collocation and field points, respectively; W_{ik} and P_{ik} are fundamental solutions for shear deformable plates [23]; $T_{\theta\alpha}$ and $U_{\theta\alpha}$ are the fundamental solutions for plane stress [11]; n_α is the unity vector normal to the boundary at field point. $\mathbf{x}' \in \Gamma$ are source points and $\mathbf{x} \in \Gamma$ and $\mathbf{X} \in \Omega$ represent field points. The value of $c_{ij}(\mathbf{x}')$ is equal to $\frac{1}{2}\delta_{ij}$ when $\mathbf{x}'$ is located on a smooth boundary. These equations represent five integral equations, the first two in (4) ($i = \alpha = 1, 2$) are for rotations, the third ($i = 3$) is for the out-of-plane displacement and two in (5) ($\alpha = 1, 2$) for in-plane displacements, which can be used to solve shear deformable plate shallow shell bending problems.