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# A theoretical formulation for the stress analysis of multi-segmented spherical shells for high-volume liquid containment

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ABSTRACT

A linear-elastic theoretical formulation is presented for the complete determination of the state of stress in large thin-walled liquid-filled vessels in the form of multi-segmented spherical shells. The transfer of membrane forces between adjacent shell segments is such that only vertical equilibrium of stress resultants needs to be preserved. The edge effect in the vicinity of the shell junctions is quantified on the basis of an approximate but accurate bending theory for spherical shells. The effectiveness of the developed formulation is demonstrated by consideration of a numerical example. Agreement with the results of finite-element modelling is excellent, showing that the presented theoretical formulation is a reliable, computationally efficient and accurate means of obtaining stresses in large multi-segmented spherical vessels.



## 1. Introduction

Thin synclastic shells of revolution find widespread application in the storage of liquids [1], on account of the structural efficiency of shells of double curvature, which allows very thin shells to resist relatively large hydrostatic pressures without rupture. Containment shells of double curvature come in a variety of shapes, from spherical, ellipsoidal, toroidal and other basic mathematical profiles, to combinations of these profiles, giving an almost limitless range of possibilities. The construction may be in thin metal, or in prestressed concrete. However, where compressive stresses exist, these structures are vulnerable to local buckling on account of the thin-ness of the shell, particularly in the case of metal construction. The thickness of the shell may be enhanced in such zones to counter any tendencies for local buckling, or stiffeners may be added to the basic shell.

Fig. 1 shows a novel form of construction for high-capacity liquid-storage vessels. The construction consists of an assembly of spherical shell segments of different radii, whose centres of curvatures all lie on the axis of revolution of the vessel taken as a whole. Thus the segments are axisymmetric in shape, where the uppermost segment is actually a cap, and successive lower segments are typically spherical frusta. Let us denote the various shells regions or segments, from top to bottom, by S1, S2, S3 and so forth. The junctions between these shell segments are denoted by J1, J2, J3 and so forth. The radii of shell S1, S2, S3, etc. are denoted by $a_1, a_2, a_3$ and so forth. As is usual for shells of revolution, the angular coordinate $\phi$ (which is the angle between the normal to the shell midsurface at any given point, and the axis of revolution of the shell assembly) is used to define the position of any point on the shell. For the shell cap S1 (uppermost portion of the assembly), the angular coordinate of the edge of the cap is denoted by $\phi_{10}$. For all other segments *Si* ($i = 2, 3, 4$, etc.) below this, the upper and lower edges of segment *Si* are defined by the coordinates $\phi_{i1}$ and $\phi_{i2}$ respectively.

Starting from the central segment (S4 in our illustration), the addition of segments S3, S2 and S1, with slope enhancements of $(\phi_{32} - \phi_{41})$, $(\phi_{22} - \phi_{31})$ and $(\phi_{10} - \phi_{21})$ at junctions J3, J2 and J1 respectively, adds height and additional storage capacity to the basic spherical vessel of radius $a_4$. Similar enhancements in capacity are also achieved by the addition of segments S5, S6 and S7 in the lower part of the vessel. The overall result is a spherical assembly of relatively large storage capacity. It is interesting to note that if this was a pressure vessel, to achieve the same storage capacity while keeping the vessel of constant radius would require a sphere of radius bigger than $a_4$, which would attract higher shell stresses (these are proportional to the radius), but in the present case of liquid containment, the stress-reducing benefit of radius limitation

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through segmental construction is offset by the higher hydrostatic pressures associated with the taller segmented vessel.

The slope and curvature discontinuities at the shell junctions attract bending disturbances [2–5], but the inward pointing kinks in the profile could have the effect of stiffening the shell response there, resulting in a beneficial lowering of hoop stresses. From an aesthetics point of view, the "lobed" or segmented geometry of the storage vessel has a pleasing appearance, which might favour the adoption of this type of vessel in locations where appearance is a major consideration.

The membrane theory of axisymmetrically loaded shells of revolution is quite appropriate for the calculation of the linear elastic response of the shell under internal hydrostatic pressure. This theory assumes there is no bending in the shell. However, and as is well-known [2–5], the membrane theory becomes inadequate in the vicinity of geometric discontinuities of the type J1, J2, J3, etc., and the more comprehensive bending theory of shells must be invoked.

A useful approach is to regard the membrane solution (for the applied surface loading on the shell) as an approximate particular solution of the differential equations describing the behaviour of the shell, and a bending correction (system of axisymmetric bending moments and shearing forces applied along the shell edge) as the homogeneous solution [2–5]; the net response of the shell is then obtained as the sum of the membrane solution and the bending correction (or edge effect). Several bending theories of varying degrees of complexity have been proposed for determining the state of stress in shells of revolution [2,3]. However, it is important to select an approach that is amenable to practical computations, and that is sufficiently accurate.

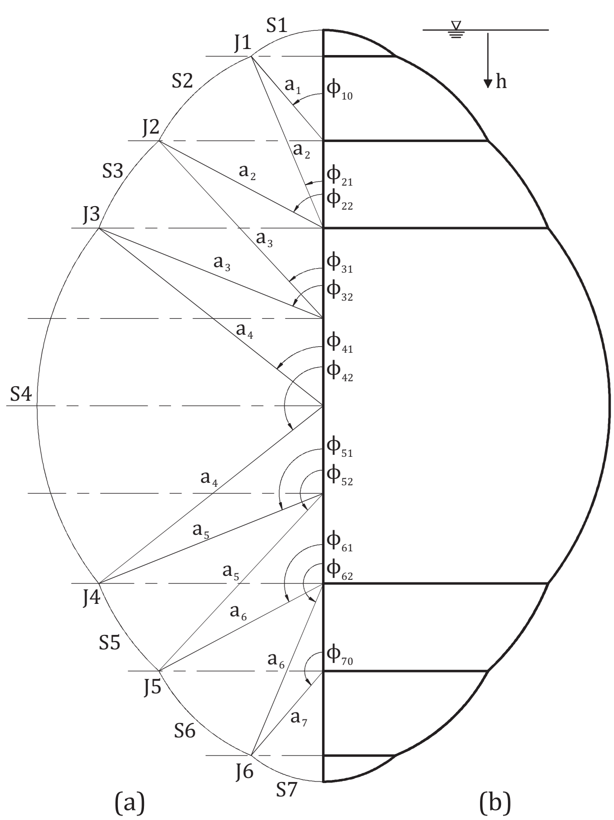


**Fig. 1.** Multi-segmented spherical vessel: (a) shell segments, junctions and corresponding geometric parameters; (b) external appearance of the vessel.

Not many analytical studies on the stress and deformation behaviour of liquid-containment shells of revolution are being reported in the literature nowadays, largely due to the fact that the Finite Element Method (FEM) has become the preferred method for investigating shell behaviour [6–8], owing to its versatility in handling irregular features of the structure, and in modelling non-linear behaviour. However, the analytical approach can still be extremely useful in those instances where the behaviour of the shell is essentially linear, and convenient mathematical solutions of the differential equations (governing shell behaviour) exist. The analytical approach, where possible, has the advantage of providing stress information without the need for potentially expensive numerical modelling, and shedding deeper insights into the behaviour of the shell simply by studying the form of the mathematical solutions. Once the analytical results are there, they may be treated as formulae, ready to be directly applied to other similar problems. Analytical solutions are also vital in checking FEM results.

Where smoothness conditions prevail, the membrane solution on its own can be a very useful tool for exploring the state of stress in liquid containment vessels in the form of arbitrary shells of revolution [9], or unusual shapes such as the triaxial ellipsoid [10]. However, where the shell geometry features discontinuities, such as sudden changes in shell thickness [11,12], a more general formulation accounting for bending effects clearly has to be employed.

Looking at the more recent literature on liquid-containment vessels, we find that cylindrical steel tanks have been studied the most. Wind-induced buckling of cylindrical tanks has received a considerable amount of attention [13,14]; such tanks are particularly vulnerable when they are empty (the presence of liquid tends to stabilise the shell against the effects of the wind). Tanks that are in close proximity of each other attract additional problems of wind interference, a phenomenon that has been the subject of some very recent studies [15,16]. Other studies have considered the response of liquid-storage cylindrical tanks to seismic excitation [17–19]. The collapse behaviour of large cylindrical steel tanks has also received attention [20], to provide a better understanding of ultimate limit-state design of such vessels. Much of the research on metal containment shells has now been codified [21,22].

At wastewater treatment works, egg-shaped digesters (with their smoothly varying geometry) offer a solution that is superior to cylindrical tanks and more conducive to the efficient mixing of sludge. The stresses and deformations in egg-shaped vessels have been investigated on the basis of the membrane theory and a simplified bending theory for shells of revolution [23,24]. A novel form of sludge digester in the form of a parabolic ogival shell has also been proposed [25], and investigated on the basis of a membrane-theory formulation, leading to some interesting insights on the behaviour of this shell form, and a set of design recommendations.

After cylindrical steel tanks, conical steel tanks come second in having been studied the most, on account of their ease of fabrication. Most previous studies on conical tanks have either sought to understand stability behaviour [26], or to develop appropriate design procedures [27]. Other studies have concentrated on understanding shell-junction effects [28,29], or the effects of external pressure [30]. The stability of vessels in the form of conical-cylindrical assemblies is a subject that has also received a considerable amount of attention [31,32].

For horizontal tanks, deviations from the normal cylindrical shape have been shown to offer enhanced structural efficiencies in comparison with conventional profiles [33]. Another class of vessel that has been studied, albeit to a lesser extent, is that of toroidal tanks [34], which find application for the storage of liquefied petroleum gas (LPG), among others. Depending on the type of cross-section chosen for the toroid, the stress distribution and