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Volume preserving projection filters and continuation methods in topology optimization

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ABSTRACT

Performance of continuation methods together with local filter operators in context of the density-based formulations for topology optimization problems is studied. In order to obtain binary discrete topologies, the volume preserving Heaviside filter is used together with continuation schemes on material penalization coefficient and filter parameters. Modifications to dual sequential optimization algorithms are presented to handle the increased non-convexity with Heaviside projection filter. Various continuations schemes are studied on test cases that include large-scale minimum compliance and compliant mechanism design problems. Finally, a continuation scheme is presented that yields good results in terms of both binary discreteness and optimal performance.

1. Introduction

In the last two decades a number of topology optimization methods have been developed and among these density-based formulations [1–4], phase-field approaches [5,6], level set methods [7] and evolutionary approaches [8–10] are some of the well-studied methods. Density-based topology optimization methods are perhaps the most commonly used methods, where nodal or element densities are considered as design variables, and the topology is defined by density distribution of two or more phases of material where the void phase is usually represented by zero density. Local filter operators are typically used in such density-based formulations: the filters ameliorates problems associated with the checkerboards and other related instabilities, and in addition, provides an intrinsic length scale which regularizes the problem formulation and make the topology results mesh independent [11]. Filters are typically used in conjunction with continuation methods on material penalization parameters to obtain results that are discrete and manufacturable.

In the context of mesh independent filters various filtering schemes have been introduced including sensitivity filters [11], density filters [12,13] and Helmholtz filters [14], among others. Among these, sensitivity filters and density filters are the most commonly used filters. In the density filter approach auxiliary variables termed density variables are introduced and the exact design sensitivities are calculated using chain rule for a given filter. On the other hand, in the sensitivity filter approach the design sensitivities are heuristically modified. Detailed description of various filtering approaches can be found in Refs. [14,15]. Density filters may not necessarily result in binary discrete designs due to the boundary diffusion effects [14,16]. To achieve this goal the study in Ref. [17] combined the density filter with an additional local filter termed Heaviside projection filter to further penalized intermediate densities so that binary discrete designs can be obtained. This filter has been used in various applications and is shown to yield binary discrete designs [18]. In the Heaviside projection filter, the curvature of the filter is controlled using a filter parameter and this filter approaches the Heaviside function as this parameter is increased. The Heaviside filter is typically used in conjunction with continuation schemes on this filter parameter wherein the filter slowly approaches the Heaviside function. A potential drawback of this scheme is that the constraints may be violated resulting in oscillations in optimization iterations and in poor performance of the optimizer as shown in Ref. [16]. To ameliorate the oscillation issues associated with the original Heaviside projection filter a volume preserving Heaviside filter is introduced in Ref. [16]. Furthermore, the study in Ref. [18] introduced modification to the optimization algorithm that eliminates the need for continuation of filter parameters in the Heaviside projection scheme. However, the effectiveness of this scheme in terms of consistently obtaining discrete optimal designs is not well established.

Although the volume preserving Heaviside filter can be used with or without continuation schemes, the benefits and drawbacks of this filter together with various continuation methods is unclear.

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For instance, it is not evident which method will lead to a better topology in terms of both binary discreteness and performance of the final optimal design. In this investigation, the performance of volume preserving Heaviside filters in conjunction with various continuation methods that can be used for obtaining binary discrete topologies is evaluated. A new continuation method is also proposed that is shown to yield overall better designs. The use of Heaviside filter leads to highly non-convex objective function and to solve the resulting optimization problems dual sequential optimization algorithms are employed. Finally, modifications to dual sequential optimization algorithms to handle the increased non-convexity of the objective function are also presented. The paper is organized as follows: in Section 2 the problem formulation is presented together with the filtering methods and convexity of the problem is evaluated. In Section 3 various continuation schemes are discussed. Dual sequential optimization methods that are used in this study are presented in Section 4 and the test cases are shown in Section 5. Finally, the important conclusions are presented in Section 6.

2. Problem formulation

In finite element setting, the topology optimization – minimum compliance and compliant mechanism design – problems are defined as follows:

$$\begin{aligned}
&\min_{\boldsymbol{x}} f_0(\boldsymbol{x}) = \boldsymbol{L}^T\boldsymbol{u} \\
&\text{Subject to}: \\
&f_1(\boldsymbol{x}) = \frac{1}{V_0}\left(\textstyle\sum_{i=1}^{n} \tilde{x}_i v_i\right) - V_\mathrm{f} \leqslant 0 \qquad (1)\\
&x_i = \phi_i(\boldsymbol{x}) \in \mathcal{B} = \{0 \leqslant x_i \leqslant 1\},\ i = 1, 2, \ldots, n \\
&\boldsymbol{K}(\tilde{\boldsymbol{x}})\boldsymbol{u}(\tilde{\boldsymbol{x}}) = \boldsymbol{P}
\end{aligned}$$

where $x_i \in [0, 1]$, $i = 1, 2, \ldots, n$ are the design variables; $\tilde{x}_i = \phi_i(\boldsymbol{x})$ are the auxiliary filtered variables that are also referred to as "physical density" variables; ϕ_i is a (local) filter operator (Fig. 1(a)); v_i is the volume of an element; V_0 is the total volume of the domain considered; and V_f is the target volume fraction. The system is also subjected to equilibrium constraints: $\boldsymbol{K}(\tilde{\boldsymbol{x}})\boldsymbol{u}(\tilde{\boldsymbol{x}}) = \boldsymbol{P}$, where $\boldsymbol{K}$ is the global stiffness matrix, $\boldsymbol{P}$ is the applied load vector and $\boldsymbol{u}$ is the corresponding displacement vector. There are n elements within the domain and each element has a density $\tilde{x}_i \in [0, 1]$. This formulation is typically termed as density filter formulation and differs from the sensitivity filter formulations that are also sometimes used in topology optimization [15].

In the above density filter formulation, the physical density variables, $\tilde{x}_i$, are used for evaluating the stiffness ($\boldsymbol{K}(\tilde{\boldsymbol{x}})$), and therefore, represents physical density. These variables are also used for plotting the final topologies. Moreover, it is important to note that the volume constraint function f_1 in Eq. (1) is written in terms of the physical density variables $\tilde{x}_i$. For solving the constrained optimization problem in Eq. (1) a nested formulation is adopted [19], wherein at each step the displacement is obtained by solving the equilibrium conditions: $\boldsymbol{K}(\tilde{\boldsymbol{x}})\boldsymbol{u}(\tilde{\boldsymbol{x}}) = \boldsymbol{P}$. Note that for minimum compliance problems $\boldsymbol{L} = \boldsymbol{P}$, whereas for compliant mechanism design problems the vector $\boldsymbol{L}$ is all zero except for the degree of freedom corresponding to the output-port where it is taken as 1.

2.1. Filters

Two desirable features in any density-based topology optimization method are: (a) the topological features (e.g. minimum member size) in the design should be explicitly controlled; and (b) the designs should be discrete (binary solutions) with minimum intermediate densities so that the final designs are manufacturable. Restriction methods such as perimeter control [20,21], global and local gradient constraints [11,22–24], and mesh independent filters [15] are typically used for obtaining mesh independent results and for controlling minimum feature size. Among these methods, the mesh independent filters are the most popular methods as they are simple and computationally efficient. In addition, they also prevent pathologies associated with checkerboards when using lower order elements [11]. These filters are typically used in conjunction with material interpolations approaches where the intermediate densities are implicitly penalized [1,25].

A local density filter operator can be considered as a mapping from the design variable space ($\boldsymbol{x}$) to the density variable space ($\tilde{\boldsymbol{x}}$): i.e. $\boldsymbol{x} \mapsto \tilde{\boldsymbol{x}}$, and the mapping is carried out via local filter operator (ϕ_i) (Fig. 1(a)). Two filtering schemes are considered in this investigation: (a) commonly used density filter termed hat filter (H-filter) [12,13], and (b) a volume preserving Heaviside filter (HE-filter) [16]. The hat filter is defined by Eq. (2):

$$\text{H-Filter}: \ \tilde{x}_i = \phi_i^t(\boldsymbol{x}) = \frac{1}{\sum_{k\in N_i} w_k}\left(\textstyle\sum_{k\in N_i} w_k x_k\right) \qquad (2)$$

Thus the mapping $\boldsymbol{x} \mapsto \tilde{\boldsymbol{x}}$ in the H-filter is carried out via local filter operator ϕ_i^t. In the above filter definition, N_i is an index set of local neighborhood the i^{th} element and is given by Eq. (3) which is based on Fig. 1(a):

$$N_i = \{k \mid \ ||\boldsymbol{X}_k - \boldsymbol{X}_i|| \leqslant R\} \qquad (3)$$

where R is the prescribed filter radius, $\boldsymbol{X}_k$ and $\boldsymbol{X}_i$ are the positions vector of centroid of the finite elements. The filter weights can be calculated using various methods [15]; in this study the weights are obtained using a linearly decaying function and is given by Eq. (4) which is based on Fig. 1(b):

$$w_k = 1 - \frac{||\boldsymbol{X}_k - \boldsymbol{X}_i||}{R} \qquad (4)$$

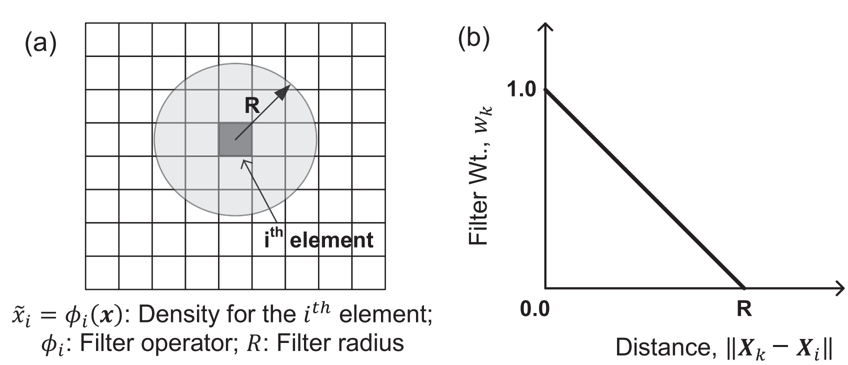

Fig. 1. Local filter operator in a density based filtering scheme.