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An efficient computational procedure for the dynamic analysis of liquid storage tanks

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ABSTRACT

A computationally efficient numerical model is developed in this study for evaluating the dynamic behavior of liquid storage tanks. This model has higher complexity than the Housner model (which corresponds to the simplest and most popular approach for approximating the behavior of rectangular and circular tanks) but still enjoys high computational simplicity to facilitate implementation in practice, while it is applicable to virtually any kind of tank geometry, providing at the same time a high degree of accuracy. In the proposed model, the liquid is assumed to be inviscid, incompressible and irrotational, and its motion is completely characterized by a velocity potential function. Thus, the Continuity and Equilibrium equations characterizing this motion take the form of Laplace and Bernoulli equations, respectively. The Laplace equation is solved through a 2D finite element scheme, and is then combined with the Bernoulli equation through the velocity potential function condensed at the free surface of the liquid. Numerical details for the practical implementation of the proposed scheme are discussed, whereas the approximation is shown to provide results with high accuracy for the dynamic behavior of different type of tanks when compared to the Housner model and a full finite element implementation. As shown in the examples considered the computational efficiency of the proposed model is such that extensive parametric studies can be performed with small numerical effort, which in turn makes the proposed model very attractive not only for analysis purposes but also for the design of liquid storage tanks and other related devices such as tuned liquid dampers.

1. Introduction

The study of the dynamic behavior on liquid storage tanks has gained significant attention in the last years as the seismic vulnerability of these tanks represents a potential source of significant economic loss due to structural failures, leakages or environmental accidents (caused by the liquid spilled out) [1–3], whereas such tanks have been proposed to be used as mass dampers to mitigate the vibration induced by wind or seismic excitations [4]. Several approaches have been proposed in this setting to model the dynamical behavior of such liquid storage tanks. In earlier studies, the fluid was taken into account by adding a mass to the structure, with characteristics computed by an analytical solution based on simplified geometries [5]. Later, Housner developed an analysis and design procedure, primarily for cylindrical and rectangular storage tanks, based on a simple mechanical model (combination of mass-spring systems with different characteristics) that represents the fluid. The computation of the physical constants in this procedure is based on the separation of the hydrodynamic behavior into two components: (1) the impulsive component that is related to the mass that moves together with the structure; and (2) the convective component that takes into account the free surface oscillations [6,7]. This is a broadly adopted model in civil engineering since it provides closed form solutions for the transmitted force due to the liquid sloshing, and represents the basis of many design codes, i.e. API 650 [8], AWWA D100 [9] and the New Zealand recommendation guidelines NZSEE [10], that establish procedures for the seismic response analysis of liquid tanks based on this linear model proposed by Housner. At the same time, it is an approximation that is based on the assumption that simplified flows can represent the actual fluid movement, restricting its use to tanks with simple geometries (such as rectangular or circular tanks).

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To obtain higher accuracy solutions, various high-fidelity procedures [11–13] have been also established, applicable to arbitrary tank geometries, using the Finite Element Method (FEM) to model the fluid utilizing (depending on the numerical scheme) displacement, pressure or potential variables to characterize the fluid motion. For generalized fluid–structure interaction applications, as the equations of the structure are expressed in terms of displacements it is convenient to also express the fluid equations through displacement variables. With respect to the fluid motion modeling, such a FEM approach, based on a displacement formulation, leads to a symmetric eigenvalue problem but it produces non-zero spurious frequencies that are difficult to identify [12,14] and suppress [13,15]. Additionally, this approach requires discretizing a vector field (displacement) instead of a scalar field (pressure or potential variables), increasing the number of degrees of freedom. In contrast, a FEM approach based on pressure or potential variables involves fewer unknowns, increasing the computational efficiency and avoiding physical inconsistencies [16]. In this case, the fluid–structure system leads to a non-symmetric eigenvalue problem, though it is possible to keep the symmetry if the fluid is described in a redundant way using both pressure and potential variables [17–19]. In particular, Olson and Bathe presented such a linear formulation based on velocity potentials and pressures [17], expanded later to take also into account gravity loads [20]. An important aspect of this formulation is its suitability for both time-history and frequency analysis of fluids with free surface. More recently, an increased number of even more complex procedures have been proposed, for example taking into account nonlinear sloshing due to large free surface motions [21–24] and including identification of damping effects introduced at the tank walls due to viscosity effects in the thin interface layer [25]. However, implementation of these procedures is almost exclusively relegated to scientific and research professional environments due to the complexity of the formulations and the high level of expertise required for their implementation.

Despite such high-fidelity modeling developments and advances in computer and computational science, the philosophy of the analysis methods of design codes is still based on analytical expressions and equivalent mechanical models. Though undoubtedly some practitioners are utilizing commercial software to solve multi-physic problems under seismic loads, avoiding the use of simplified models proposed by the design codes or even the complex implementation of the procedures described above, such approaches are still not widely used (presumably because a significant background is required not only in the software know-how but also in the theoretical knowledge about the involved physics). Furthermore, many traditional software packages used for seismic and structural analysis lack fluid–structure interaction modules, enforcing engineers to work with alternative packages that were not designed to perform seismic analysis. There is a gap for a methodology that is more simple and attractive than the commercial packages but still maintains the accuracy of the advanced methods presented in the literature.

Motivated by this realization, the main novel contribution of this work is to develop a simplified, computationally efficient framework, utilizing a FEM modeling based on potential variables and a static condensation approach while assuming the tank walls as rigid, for describing the dynamic behavior of arbitrary geometry liquid storage tanks under seismic excitation. This approach can facilitate a computationally efficient description of the dynamic behavior of tanks (supporting frequency and time domain analysis as well as eigenvalue analysis) including its interaction with a supporting structure (as needed for TLD design applications), though it cannot provide detailed predictions for localized failure phenomena related to the tank walls (which are considered rigid). The proposed numerical procedure expresses the linear sloshing problem as a second order linear system of equations, where the independent variables are the vertical elevation of the free surface and the excitation is directly related to the ground acceleration. The fluid is assumed ideal while the tank walls and bottom are assumed rigid. As the fluid is considered ideal, it is possible to adopt a FEM formulation based on potential variables, reducing the number of unknowns and avoiding problems with spurious frequencies. The rigid tank assumption simplifies the fluid–structure coupling since it is not necessary to generate a mesh for the tank walls and bottom and to match it with the fluid mesh. In this sense, the procedure is easier to implement than the ones cited previously [12–20]. Furthermore, the proposed procedure allows studying the sloshing effect over the tank support rather than the sloshing effect over the tank itself [12–20]. Although the methodology is standard, the numerical procedure offers significant advances and physical insight as: (1) the equations are expressed in terms of physical variables (free surface elevation and ground acceleration), (2) the system of equations is similar to that of mass-spring systems, (3) the approach is valid for any tank geometry, (4) the formulation is suitable for both time-history and frequency analysis, (5) it allows for a straightforward coupling between rigid tanks and elastic structures, (6) the procedure is relative easy to implement or understand, and (7) it offers significant advances over the models suggested by the design codes. Ultimately, the proposed numerical procedure, from now on denoted as Simplified Sloshing Model (SSM), enjoys such computational simplicity and efficiency that it can be used for various tasks such as parametric studies, preliminary dimensioning of tanks, seismic performance identification, or even design and dimensioning of tuned liquid dampers.

2. Description of the Simplified Sloshing Model (SSM)

A 2D schematic diagram of a liquid storage tank is presented in Fig. 1. It is important to mention that, although the formulation of the proposed Simplified Sloshing Model could be either 2D or 3D, this paper presents in detail only a 2D formulation because it is the case most extensively studied in the literature, and the validation will be made considering several 2D existing examples analyzed independently by other authors. An inertial system of reference x–z is located at the middle of the non-perturbed free surface and an auxiliary coordinate η is defined to measure the relative displacement between the free surface and the coordinate system. Let Ω represent the volume of liquid, Γ_o the non-perturbed free surface (at $z = 0$), Γ_s the free surface at any time t, and Γ_p the walls and bottom surfaces (all these variables are also shown in Fig. 1). The liquid motion is modeled using principles of Mass and Momentum Conservation, while the tank walls and bottom are considered to be rigid. The liquid is assumed to be inviscid, incompressible, and irrotational, allowing its motion to be completely defined by a velocity potential function φ. Additionally, body forces are assumed conservative and nonlinear terms are

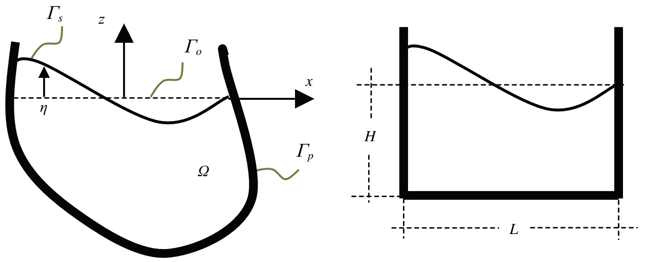

Fig. 1. Scheme of the liquid storage tank: (left) arbitrary geometry; (right) rectangular.