Engineering Structures 85 (2015) 277–292

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Buckling and postbuckling behavior of shear deformable anisotropic laminated beams with initial geometric imperfections subjected to axial compression

Zhi-Min Li [a,c], Pizhong Qiao [b,c,*]

[a] State Key Laboratory of Mechanical System and Vibration, Shanghai Key Lab of Digital Manufacture for Thin-Walled Structures, School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai 200240, PR China
[b] State Key Laboratory of Ocean Engineering and School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai 200240, PR China
[c] Department of Civil and Environmental Engineering, Washington State University, Pullman, WA 99164-2910, USA

ARTICLE INFO

Article history:
Received 10 May 2014
Revised 13 December 2014
Accepted 15 December 2014
Available online 8 January 2015

Keywords:
Anisotropic laminated composite beams
Higher-order shear deformation beam theory
Buckling
Postbuckling
Axial compression
Imperfection

ABSTRACT

Buckling and postbuckling behavior of shear deformable anisotropic laminated composite beams with initial imperfection subjected to axial compression is presented. The material in each layer of beams is assumed to be linearly elastic, anisotropic and fiber-reinforced. The governing equations are based on the higher order shear deformation beam theory with a von Kármán-type of kinematic nonlinearity. Composite beams with the fixed–fixed, fixed–hinged, and hinged–hinged boundary conditions are considered. A generic imperfection function for one-dimensional composite beams is adopted to model various possible initial geometric (e.g., sine, local, and global type) imperfections. The nonlinear prebuckling deformation and initial geometric imperfection of the beam are both taken into account. A numerical solution of nonlinear partial–integral differential form in terms of the transverse deflection is employed to determine the buckling load and postbuckling equilibrium path of composite beams. The results obtained by combining the Newton's iterative method with the Galerkin's method are theoretically exact from the transverse and longitudinal displacements for anisotropic laminated beams under the axial compressive loads using the secondary parameter conversion technique, and they are validated by comparing with those available in the literature. The numerical illustrations are presented for the postbuckling response of laminated beams with different types of boundary conditions, ply arrangements (layups), geometric and physical properties. The results reveal that the geometric and physical properties and boundary conditions have a significant effect on postbuckling behavior of anisotropic laminated composite beams.

1. Introduction

Composite structures, like beams and plates, are broadly used in various engineering applications, such as airplane wings, helicopter blades as well as many others in the aerospace, mechanical, and civil industries. Due to the outstanding engineering properties, such as high strength/stiffness to weight ratios, the laminated composite beams are likely to play a remarkable role in the design of various engineering type structures and partially replace the conventional isotropic beam structures. Interest in the structural buckling and postbuckling analysis of anisotropic composite laminated beams has led to a need for more accurate analysis especially in the case of critical structures.

Many studies have observed the buckling and postbuckling behavior of beam-type structures, and numerous attempts have been made to predict such phenomenon for isotropic or orthotropic beams. Theories of beams involve basically the reduction of a three dimensional problem of elasticity theory to a one-dimensional problem. Since the thickness dimension is much smaller than the longitudinal dimension, it is possible to approximate the distribution of the displacement, strain and stress components in the thickness dimension. Based on the assumption that the axial line of the beam is inextensible, Timoshenko and Gere [1] examined the postbuckling of compressed beam clamped at one end and free at the other end and investigated it using the exact expression for the curvature in the differential equation of the deflection curve. Comer and Levy [2] proved that the inflatable

* Corresponding author at: Department of Engineering Mechanics, Shanghai Jiao Tong University, Shanghai 200240, PR China.

E-mail addresses: qiao@sjtu.edu.cn, qiao@wsu.edu (P. Qiao).

http://dx.doi.org/10.1016/j.engstruct.2014.12.028
0141-0296/

beams can be considered as the usual Euler–Bernoulli beams. The theory is applicable to slender beams and should not be applied to thick or deep beams since it is based on the assumption that the plane sections perpendicular to the neutral layer before bending remain plane and are perpendicular to the neutral layer after bending, implying that the transverse shear and transverse normal strains are zero. Wang [3] dealt with the buckling of the axial compressive beams with the pinned–fixed ends using a shooting method as well as a perturbation method. Torkamani et al. [4] and Hori and Sasagawa [5] used the approximate second-order analysis, such as the finite element method with large deflections and with or without large strains. Li and Batra [6] investigated the analytical relations between the critical buckling load of a functionally graded material (FGM) Timoshenko beam and that of the corresponding homogeneous Euler–Bernoulli beam subjected to axial compressive load with simply supported, clamped and clamped–free boundaries. Vo and Thai [7] developed a one-dimensional displacement-based finite element method to accurately predict the critical buckling loads of rectangular composite beams with the corresponding mode shapes for various configurations. Furthermore, Chia et al. [8] reported the structural responses of generally-laminated composite columns subjected to uni-axial compression and transverse load, and the closed-form expressions were developed and presented to analyze the buckling and bending responses of generally-laminated composite beams with various boundary supports based on the Euler–Bernoulli beam and classical lamination theories. Emam and Nayfeh [9] and Nayfeh and Emam [10] obtained an exact solution for the postbuckling behavior of the composite beams with fixed–fixed, fixed–hinged and hinged–hinged boundary conditions based on the Euler–Bernoulli beam theory. Emam [11] studied the static and dynamic behavior of geometrically-imperfect laminated composite beams, and as a result, its lateral deflection is obtained as a function of the applied axial load, a parameter designating the laminate, and imperfection. Using the Rayleigh–Ritz method, Gupta et al. [12] predicted the postbuckling behavior of composite beams and compared their solution with the results obtained from the finite element analysis for general lay-up. Khdeir and Reddy [13] studied the buckling behavior of cross-ply laminated beams with arbitrary boundary conditions. The classical, first-, second- and third-order shear deformation theories were used in the analysis. Khamlichi et al. [14] presented the different formulations to the solution of the large-deflection problem of a hinged–hinged elastic bar with outside sway under axial compressive load, and they discussed the effects of the axial strains and shear deformations using the asymptotic expansion technique on the postbuckling behavior. Matsunaga [15] analyzed the buckling stresses of the laminated composite beams by taking into account the complete effects of transverse shear, normal stress and rotary inertia. Aydogdu [16] performed the buckling analysis of cross-ply laminated beams subjected to different sets of boundary conditions by using the Ritz method. The analysis was based on a three-degree-of-freedom shear deformable beam theory. It is found that the postbuckling response increases as the shear deformation becomes more significant. The elasticity solutions for plates of rectangular cross sections were given by Pagano [17,18] by comparing the solutions of several specific boundary value problems in his theory to the corresponding ones of elasticity solutions. In general, it is found that the conventional plate theory leads to a very poor description of laminate response at low span-to-depth ratios, but it converges to the exact solution as this ratio increases. While the cylindrical bending provides a convenient tool for performing a one-dimensional analysis of laminated plates, a theory for anisotropic laminated beams is also important. The difference between the cylindrical bending and beam bending is analogous to the difference between the plane strain and plane stress in classical theory of elasticity. The major difference is in the term of bending stiffness. Recently, Foraboschi [19,20] developed the analytical modeling within the framework of one-dimensional elasticity. This new approach found the exact solution for the laminated composite beams and laminated glass columns. Moreover, the analytical modeling was recently developed within the framework of two-dimensional elasticity [21–24]. The classical and first-order shear deformation theories underestimate the amplitude of buckling while the considered higher-order theories yield very close results. Murthy et al. [25] developed a refined 2-node, 4 degree-of-freedom node beam element based on a higher-order shear deformation theory for the axial flexural shear coupled deformation in asymmetrically-stacked composite beams. In spite of the availability of finite element method and powerful computer programs, the second- or higher-order analysis of a composite beam is still an impractical task to most structural designers due to the limitation of the number of degrees of freedom (DOF) required to achieve a desired level of precision and efficiency. The use of elasticity theory is practically unfeasible due to mathematical difficulties and the complexity of laminated systems. This led to the development of refined shear deformation theories for beams which approximate the two dimensional solutions with reasonable accuracy.

Most recently, Carrera et al. [26-30] and Giunta et al. [31,32] established the Carrera Unified Formulation (CUF) which has hierarchical properties and is capable of dealing with most typical engineering challenges, i.e., the error can be reduced by increasing the number of the unknown variables. It overcomes the problem of classical formulae that require different equations for tension, bending, shear, and torsion, and it can be applied to any beam geometries and loading conditions, reaching a high level of accuracy with low computational cost. It can tackle problems that in most cases are solved by employing the plate/shell and 3D formulations. The comparison of the analytical results with the experimental results shows good correlation in general.

Challamel et al. [33] investigated the buckling behavior of generic higher-order shear deformable beam models in a unified framework. Buckling solutions were presented for usual archetypal boundary conditions, such as pinned–pinned, clamped–free, clamped–hinge, and clamped–clamped boundary conditions. The results were then extended to general boundary conditions based on the generalized linear elastic connection law including the vertical and rotational stiffness boundary conditions. In addition, Emam [34] studied the postbuckling behavior of symmetrically-laminated and simply-supported beams by solving the nonlinear governing equations for which the critical buckling load is obtained by solving their linear counterpart. The results showed that the shear deformation of moderately-thick beams or beams made up of highly anisotropic materials has a significant effect on the postbuckling behavior of laminated composite beams.

Moreover, Ghugal and Shimpi [35] presented a review of refined shear deformation theories for the structural analysis of shear deformable isotropic and laminated beams and on the recent advances in the modeling and analysis of laminated beams. Mullapudi and Ayoub [36] analyzed the reinforced concrete columns subjected to combined axial, flexure, shear, and torsional loads. Wang et al. [37] illustrated how the shear deformation theories provide accurate solutions when compared to the classical theory for both the beams and plates. Waas [38] presented an asymptotic initial postbuckling analysis of pinned–pinned and clamped laminated beams, incorporating the first-order shear deformation effects.

The imperfection sensitivity of the beams has been extensively studied, and the maximum load carrying capacity is usually calculated as a function of normalized imperfection amplitude. In fact, the effect of initial geometric imperfection may play a great role in the postbuckling behavior of moderately-thick anisotropic