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Verification formulae for structural glass under combined variable loads

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ABSTRACT

It is well known that, depending upon the thermo-hygrometric environment, surface flaws in glass can grow over time even when they are well below the critical size, eventually leading to failure of the stressed material. This phenomenon, usually referred to as *subcritical crack growth*, or *static fatigue*, implies that the macroscopic strength of glass depends upon the characteristic duration of the applied loads. Various criteria have been proposed to evaluate the effects of the simultaneous combinations of actions applied at different times of the load history. Here, starting from a consolidated model of subcritical crack growth, an analytical approach to this problem is presented. Safety domains are calculated and compared with the approaches prescribed by recent proposals for standards. The analysis of a few case studies confirms that some approaches are not on the safe side, whereas other approaches can be too conservative. A proposal for new verification formulae is presented.

1. Introduction

Recent architectural trends have favoured the development of new technologies that have brought considerable improvements in the use of glass in buildings. This material has become a real structural material, not any more confined to form the building envelope, but used to construct self-supporting roofs, floors, staircases, beams, pillars and frames. When the load bearing capacity becomes a basic requirement, it is customary to speak of "structural glass", even if this term may lead to ambiguities because the adjective "structural" refers to the application rather than to the material, which is nothing but the same commercial glass used elsewhere and not a special glass, *ad hoc* manufactured. Since glass has to safely withstand considerable loads, structural verifications have to be performed, but unlike other structural materials, whose properties are well-known so far, the design methods for structural glass are still the subject of studies. On-going research is redirecting them towards even more precise approaches.

The strength of glass, the brittle material *par* excellence, is in fact affected by some peculiar aspects that are not relevant for other structural materials, like steel and concrete. Certainly, glass does not exhibit any ductility at the macroscopic level, and breaks as soon as the stress at one point overcomes a certain limit. More precisely, failure of glass is governed by the existing microscopic surface flaws, which can open and progress under the applied stress [1]. Linear Elastic Fracture Mechanics (LEFM) is therefore the most useful tool to investigate the mechanical properties of glass and interpret its brittle behavior. In particular an intriguing phenomenon, common to most brittle solids and usually referred to as *slow crack propagation* or *static fatigue* [2], is that cracks can slowly grow in time even when the stress intensity factor is far below the critical limit. This produces the delayed rupture of the element when the applied macroscopic stress is constant, so that the macroscopic glass strength strongly depends upon the load history.

There are several models to evaluate the influence of the static fatigue phenomenon upon the strength of glass, among which one may cite the pioneering work of Brown [3] with his *load duration theory*. The *glass failure prediction model* by Beason and Morgan [4] is taken as the reference for standards in the United States [5] and Canada [6]. Models that interpret the subcritical crack growth with a LEFM approach are those proposed by Sedlacek [7], Fischer-Cripps and Collins [8], which have been adopted in Europe [9] and in Italy in particular [10].

At the practical design level, the semi-probabilistic approach prescribes to compare the stress induced by the design actions at the critical points with the design strength of glass, through the definition of appropriate partial safety factors [11]. Most standards [12,13,9,10,14] introduce in the expression of the design strength a "load duration factor" k_{mod}, which is a function of the characteristic duration of the action. However, during its lifetime the construction-work is subject to actions of various nature with different characteristic durations, which need to be combined to account for their

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Nomenclature

$\langle\cdot\rangle^+$	positive part of a function
α_{ij}	ratio of strengths, $\alpha_{ij} = f_{gi}/f_{gj}$
$\dot{\sigma}_t$	stress rate
γ_G	partial safety factor for self-weight
γ_Q	partial safety factor for variable actions
γ_{MA}	partial safety factor for annealed glass
γ_{Mv}	partial safety factor for pre-stressed glass
σ_a	stress induced by an applied constant load
σ_p	surface compression stress due to tempering/toughening
$\sigma_\perp$	macroscopic tensile stress normal to the crack plane
$\sigma_{\max}$	maximum stress on the glass plate
$\sigma_{\mathrm{eff},d}$	effective stress induced by the combination of applied actions
σ_j	stress due to the *j*-th applied constant load
$\Psi_{0,i}$	factors for combination value of accompanying variable actions
Ψ_1	factor for frequent value of a variable action
$\Psi_{2,i}$	factor for quasi-permanent value of a variable action
$\hat{h}_w$	deflection-effective thickness
$\hat{h}_\sigma$	stress-effective thickness
c	crack length
c_c	critical limit for a surface crack
c_f	crack length at failure
c_i	initial crack length
f_{bk}	characteristic bending strength of *pre-stressed* glass
f_c	strength associated with critical crack growth
$f_{gd,b}$	design strength of annealed glass
$f_{gd,j}$	design strength of glass for the *j*-th action
$f_{gd,p}$	additional design strength due to surface compression by tempering/toughening
f_{gd}	design strength of glass
f_{gj}	reference strength of glass for the *j*-th action
f_{gk}	Characteristic bending strength of *annealed* glass
f_g	reference tensile strength of glass measured from the test
k_v	factor for the type of tempering
$k_{mod,j}$	load duration factor for the *j*-th action
k_{mod}	load duration factor
k_{sp}	factor for the surface finishing of glass
n	static fatigue exponent
t_c	time required for failure
t_f	total duration of actions
t_j	duration of the *j*-th load
v_0	reference velocity for subcritical crack growth
$w_{\max}$	maximum deflection of the glass plate
G	value of permanent actions (self-weight)
G_{int}	shear modulus of PVB interlayer
K_{Ic}	critical stress intensity factor for mode I loading
K_I	stress intensity factor for mode I loading
L	length of the non supported edges of a glass plate
N	number of actions
$Q_{k,1}$	characteristic value of the dominant action
$Q_{k,i}$	characteristic value of the other variable actions
Q_m	maintenance load
Q_s	snow load
F_d	design value of the combination of actions
T	temperature
Y	shape factor

possible simultaneous effects. Combining these effects in a simple verification formula is not easy, because the macroscopic strength depends, through k_{mod}, upon the duration of the applied actions.

Alternative approaches have been proposed. For example, the current project of European standard prEN16612:2013 [9] indicates that "the k_{mod} for the load combination is the highest value associated with *any* of the loads in the combination". To illustrate, if the structure is subjected to self-weight, wind and snow loads, the strength of glass to be compared with the stress produced by the load combination should be that corresponding to the k_{mod} associated with the wind action, which is the highest because wind is the action with shortest characteristic duration. There is however a conflicting point in the document, because at paragraph 9.1.3 the project standard indicates that "the value of the load duration factor used to calculate the design value of strength shall be appropriate to the anticipated duration of the single load (or the dominant load where there are combined loads)". What is the *dominant load* is not clear, because it may be either the one that produces the most critical state of stress, or the one with the greatest duration, but in any case it is not necessarily the one with the highest k_{mod}. It is certain, however, that consideration of the highest k_{mod} in the combination cannot be on the safe side. Other structural recommendations, instead, consider a cumulative damage rule similar to Palmgren–Miner's, originally proposed for the fatigue of metals. Such an approach is followed by the Italian recommendations [10], according to which the stress produced by each one of the design loads is directly compared with the design strength for that load, calculated according to its duration through the corresponding coefficient k_{mod}.

However, to our knowledge, how to combine the effects of variable load histories still represents an open problem, because alternative approaches are constantly being proposed by several authors. For example, Overend [15] has suggested the use of a stress-history interaction equation which considers the effect of short term (i.e., wind action), medium term (i.e., snow action) and long term loads (i.e., self weight). This treatment was received by an old project for an European standard [16], but it seems rather conservative.

Here, the problem is approached from a theoretical point of view. Starting from the subcritical crack growth model introduced by Wiederhorn [2], which is commonly considered a reference point, the slow crack propagation consequent to simultaneous stress histories is analytically calculated, evaluating as well the macroscopic limits of failure under combined actions. Very general load histories will be considered, interpreting cases in which an arbitrary numbers of actions are in combination. The resulting safety domains are compared with the prescriptions of current structural recommendations. Indications will also be given about how to consider the beneficial effects of glass prestressing, obtained through tempering or toughening processes. Finally, an explicit formula is indicated for the verification of glass strength under combined variable loads. Two case studies will be analyzed in detail in order to illustrate the present formulation and compare it with other approaches so far proposed.

2. Static fatigue of glass

The macroscopic mechanical properties of glass derive from its brittle nature, which is characterized by a high sensitivity to stress concentrations at surface flaws. Characterization of the fracture strength of glass cannot neglect to consider subcritical propagation of such flaws.