Engineering Structures 78 (2014) 28–40

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

A critical look into Rayleigh damping forces for seismic performance assessment of inelastic structures

Pierre Jehel *

MSSMat (UMR 8579) CNRS/École Centrale Paris, Grande voie des Vignes, 92295 Châtenay-Malabry Cedex, France
Department of Civil Engineering and Engineering Mechanics, Columbia University, 630 SW Mudd, 500 West 120th Street, New York, NY 10027, USA

ARTICLE INFO

Article history:
Available online 22 August 2014

Keywords:
Seismic performance
Damping forces
Nonlinear structure
Computational mechanics
Experimental test design
Earthquake engineering

ABSTRACT

Rayleigh damping forces are commonly introduced in the numerical simulations of nonlinear structures run to assess structural performance in case of an earthquake. Their purpose is to account for energy dissipative mechanisms not otherwise explicitly represented in the model. When caused by interactions between the structure and its surrounding environment, energy dissipation is external to the structure, whereas it is internal when resulting from energy absorption mechanisms activated in the structure. In this paper, the concept of *discrepancy forces* is introduced in the framework of computational dynamics. Then, damping forces are presented as a model of these so-called discrepancy forces to represent internal energy dissipation. On the other hand, the discrepancy forces are identified from a set of experimental data recorded during shaking-table test of a ductile moment-resisting frame, which provides the rationale for a critical look into Rayleigh damping forces. It is in particular observed that, for the structure tested, the Rayleigh damping model used is inaccurate as a representation of the discrepancy forces. Besides, while the knowledge of the discrepancy forces allows for rationally discussing the capabilities of the inelastic structural model to represent the actual behavior of the structure, this is only possible to a limited extent with the Rayleigh damping model used.

1. Introduction

1.1. Seismic structural performance assessment

Seismic performance assessment of inelastic structures is a key step in a seismic risk management process that aims at mitigating the risks for the populations and the infrastructures in seismically active regions. Seismic risk is a combination of seismic hazard and structural vulnerability and can be effectively formalized and communicated in a probabilistic setting (see e.g. [1]):

$$P_{\mathrm{PL}} = \int P[\mathrm{EDP} \geqslant \mathrm{EDP}_{\mathrm{PL}} \mid \mathrm{IM} = x] \cdot P[\mathrm{IM} = x] dx. \qquad (1)$$

In this equation, PL is the performance level associated to a certain value of an engineering demand parameter (EDP) of interest. For instance, in the case of moment-resisting frame structures, maximum interstory drift is often used as the EDP of interest (see [1–3] among others) because it is possible to map it to meaningful PL such as "immediate occupancy", "structural damage" or "collapse prevention" [1]. In Eq. (1), IM refers to the intensity measure of the seismic ground motion, and the probability to observe an earthquake with an IM equal to x in the region of interest, that is $P[\mathrm{IM} = x]$, is given by seismic hazard maps. This paper focusses on the role of Rayleigh damping forces in the vulnerability assessment of nonlinear structures, that is on the computation of the conditional probability that a structural performance criteria is exceeded given a certain IM of the ground motion, when the EDPs are computed through time-history analyses.

1.2. Uncertainties in the performance assessment

Whether they are pertaining to the ground motion signal or to the structural response, uncertainties are numerous and can dramatically impact the conclusions of seismic risk analyses. To identify which of the potential uncertainty sources have to be accounted for in the communication of risk analyses results, a series of sensitivity analyses has been conducted in the structural earthquake engineering community (e.g. [2–8] among others). In particular, the studies in [4,5,7] explicitly account for Rayleigh damping as a potential contributor to the uncertainty in the EDP of interest.

* Address: MSSMat (UMR 8579) CNRS/École Centrale Paris, Grande voie des Vignes, 92295 Châtenay-Malabry Cedex, France.
E-mail address: pierre.jehel@ecp.fr

http://dx.doi.org/10.1016/j.engstruct.2014.08.003
0141-0296/

The sources of uncertainty that most strongly affect the repair cost in an earthquake have been sought in [4] using sensitivity analyses. A high-rise reinforced concrete nonductile moment-resisting frame is studied. Structural Rayleigh damping ratio is found to be a minor source of uncertainty in the adopted structural performance measure with respect to the capacity of the structural elements to damage and the seismic ground motion intensity.

In [5], the authors use FOSM method to investigate the sensitivity of a series of EDPs to uncertain parameters among which Rayleigh damping ratio. The building studied is a seven-story reinforced concrete shear-wall structure. It is concluded that, for the local EDPs considered (curvature in critical sections), viscous damping is the second most significant source of uncertainty after the intensity of the ground motion.

In [7], a sensitivity analysis of the maximum interstory drift to inelastic frame element properties, beam–column joint properties as well as structural viscous damping ratio is performed for a reinforced concrete frame structure at various seismic hazard levels. Although the uncertainty in the ground motion dominates the overall uncertainty in the interstory drift, Rayleigh damping ratio is found to be one of the most significant other contributors to the EDP of interest.

1.3. Objective and scope of the paper

On the one hand, it has been observed that Rayleigh damping can be a significant contributor to the overall uncertainty in the EDPs of interest for seismic performance assessment of inelastic structures [5,7]. On the other hand, it has been shown that using Rayleigh damping forces along with an inelastic structural model can be problematic and lead to unintended consequences that can compromise the validity of the analyses outputs [9,10]. Therefore, the objective of this paper is to provide a rational discussion on the validity of Rayleigh damping forces in the time history analyses of inelastic structures and to shed light on a potential strategy to model realistic damping forces in inelastic simulations along with improving the predictive capabilities of the structural models.

Rayleigh damping can be used in seismic simulations either to account for energy dissipation mechanisms that are external to the structure or for energy absorption mechanisms that are internal to the structure. This work focusses on internal energy absorption only. Besides, in case internal energy absorption has to be modeled, we adopt the viewpoint of Rayleigh damping forces being added to complete the seismic energy absorption capacity of the inelastic structural model. In other words, Rayleigh damping forces are not considered in this paper as intrinsic to the structural response but as some ad hoc correction of deficiencies of the inelastic structural model to accurately represent the actual structural response to the seismic action.

The approach adopted in this work is fundamentally different from what is developed in studies focused on structural system identification (see e.g. [11–13]). In these latter studies, the structure is considered as a system that modifies the seismic ground motion (input signal) into the data (e.g. displacements) recorded at monitored points (output signals). In such analyses, there is no explicit inelastic structural model used to simulate the structural response: the structural system is represented by linear differential equations characterized by modal damping ratios and frequencies that can be identified in the process. Hereafter however, an inelastic structural model is constructed to approximate the response of the structure, and the damping forces time history is identified.

1.4. Outline of the paper

The outline of the paper is as follows. In the next section, basic equations of nonlinear dynamics are first recalled as a baseline for introducing the concept of discrepancy forces in the framework of computational nonlinear dynamics. In particular, the need for experimental data to calculate these discrepancy forces is pointed out. Section 3 is devoted to a short description of the shaking table tests during which the experimental data that are used thereafter were recorded. Then, two inelastic structural models of the tested moment-resisting reinforced concrete frame are presented. They are developed using fiber frame elements and simple inelastic beam-to-column connections. The discrepancy forces are calculated for both structural models in Sections 5 and 6. How discrepancy forces can be used to improve structural models is in particular discussed and used to parameterize the improved structural model used in Section 6. Before closing the paper with some conclusions, Section 7 presents a critical discussion on Rayleigh damping forces based on the rationale provided by the knowledge of the discrepancy forces.

2. Damping forces revisited – discrepancy forces

2.1. Classical computational nonlinear dynamics

We assume that the dynamic nonlinear structural problem is cast in a standard finite element form. We also assume that displacement (hysteresis), velocity (viscosity), and acceleration-proportional (inertia) forces contribute to the structural response (left-hand side of the equation):

$$\mathbf{M}\ddot{\mathbf{u}}(t_n) + \mathbf{C}(t_n)\dot{\mathbf{u}}(t_n) + \mathbf{F}^{hys}(\mathbf{u}; t_n) = \mathbf{F}^{ext}(t_n), \tag{2}$$

where $\mathbf{M}$ and $\mathbf{C}$ are the mass and damping matrices, $\mathbf{F}^{hys}$ is the structural hysteretic restoring force vector, and $\mathbf{F}^{ext}$ is the external loading vector. $t_n \in \mathcal{T}$ with $\mathcal{T} = \{n \times \Delta t \mid n \in [0, 1, \ldots, N], \Delta t = T/N > 0\}$ is a discrete process. $\mathbf{F}^{ext}$ typically consists of the static loadings (dead and service loads), the forces induced by the seismic ground motion, and the reactions at the connections between the structure and its environment. Also, if some energy dissipation sources that are external to the structure are present in the system (structure equipped with energy dissipation device that has known physical properties), they are considered here to act as external loading, so that the viscous and hysteretic forces only account for mechanisms that are internal to the structure.

Because the structural response is possibly nonlinear, we rewrite Eq. (2) as a residual vector $\mathbf{R}$ that has to be iteratively set to zero:

$$\mathbf{R}(\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}; t_n) = \mathbf{0}. \tag{3}$$

At iteration k, with the subscript n referring to t_n, the Newton–Raphson updating residual reads

$$\mathbf{R}_n^{(k+1)} = \mathbf{R}_n^{(k)} + \left.\frac{d\mathbf{R}}{d\mathbf{u}}\right|_n^{(k)} d\mathbf{u}_n^{(k)} = \mathbf{0}, \tag{4}$$

where

$$\mathbf{R}_n^{(k)} = \mathbf{F}_n^{ext} - \mathbf{M}\ddot{\mathbf{u}}_n^{(k)} - \mathbf{C}_n\dot{\mathbf{u}}_n^{(k)} - \mathbf{F}^{hys}(\mathbf{u}_n^{(k)}) \tag{5}$$

and the total tangent matrix

$$\begin{aligned}\mathbf{S}_n^{(k)} &= -\left.\frac{d\mathbf{R}}{d\mathbf{u}}\right|_n^{(k)} = -\left.\frac{\partial\mathbf{R}}{\partial\mathbf{u}}\right|_n^{(k)} - \left.\frac{\partial\mathbf{R}}{\partial\dot{\mathbf{u}}}\right|_n^{(k)}\frac{d\dot{\mathbf{u}}}{d\mathbf{u}} - \left.\frac{\partial\mathbf{R}}{\partial\ddot{\mathbf{u}}}\right|_n^{(k)}\frac{d\ddot{\mathbf{u}}}{d\mathbf{u}} \\ &= \mathbf{K}_n^{(k)} + c_C\mathbf{C}_n^{(k)} + c_M\mathbf{M},\end{aligned} \tag{6}$$

where $\mathbf{K} = d\mathbf{F}^{hys}/d\mathbf{u}$ is the structural tangent stiffness matrix, $c_C = d\dot{\mathbf{u}}/d\mathbf{u}$ and $c_M = d\ddot{\mathbf{u}}/d\mathbf{u}$ are coefficients dependent on both the time step Δt and the parameters of any one-step time integration algorithm.