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Optimal performance-based design of non-linear stochastic dynamical RC structures subject to stationary wind excitation

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ABSTRACT

Performance-based design, or performance-based engineering (PBE), is currently well accepted as a proper methodology for assessing risk and designing facilities which can be subject to continuous levels of damage caused by extreme responses under various hazards of varying magnitudes. However, the difficulties in assessing probabilities associated with different hazard and performance levels, especially when nonlinearity is considered in dynamically excited structural systems, have been a limiting factor for incorporating PBE into design optimization. This paper advances the state-of-the-art by incorporating PBE into the optimal design of non-linear/hysteretic stochastic dynamical systems. The approach combines a statistical linearization technique with time-variant reliability analysis concepts, in order to formulate a total expected life-cycle cost optimization problem. As a numerical example, reinforced concrete buildings modeled as MDOF Bouc–Wen hysteretic systems subjected to wind excitation are studied. Optimal transversal stiffness of the buildings columns are obtained both for the linear and the nonlinear cases, as well as for various design life values. Optimal stiffness values determined herein consider the initial costs but also expected losses over the lifetime of the structure, for several wind hazard magnitudes and displacement response levels of the structure.

1. Introduction

Civil engineering structures (e.g. high rise buildings, bridges, offshore platforms) are often subject to evolutionary stochastic excitation, such as wind, earthquake and ocean waves [1,2]. In this regard, a concerted effort has been made to reduce the costs related to failure due to these kinds of excitation (e.g. [3]). One of the most recent trends is the use of Performance Based Design (PBD) or Performance Based Engineering (PBE) to assess risks associated with civil engineering structures [4,46]. Specifically, PBE is a design/analysis philosophy for addressing facilities subject to several hazards of different magnitudes for which various levels of performance (comfort, minor damage, gross damage and collapse) are accepted with different probabilities. The concept was originally developed for earthquake engineering [5–9], but was promptly extended to wind engineering [10–12] and more recently to seismic pounding [13] and hurricane hazards [14] applications.

The concept of PBE can be readily combined/associated with the idea of design optimization for minimal life-cycle costs [15–19]. In this regard, the costs associated with different expected displacement responses of the structure are evaluated over different load (hazard) intensities. The total expected life-cycle cost, including initial (construction) costs and expected losses due to different hazards, is minimized with respect to the appropriate design variables.

The combination of PBE and design optimization, however, is not a straightforward task, due to the significant difficulties in assessing probabilities associated with different hazard and structural response levels. Most PBE research papers encountered in the literature address risk analysis, but contributions combining PBE with design optimization are still scarce [9,43–45]. On the other hand, several seminal papers address optimization of stochastic dynamical systems under uncertainties [22–32], but not under a PBE perspective. Obviously, consideration of nonlinear structural models [1,2,9,27,30,32], in this context, is an added difficulty.

The present paper advances the state-of-the-art by incorporating PBE and life-cycle cost minimization in the optimal design of non-linear/hysteretic stochastic dynamical systems. A framework is developed for optimizing the performance of structural systems exhibiting complex nonlinear/hysteretic behavior, when subject to stochastic excitation. In this framework, statistical linearization

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(e.g. [33,34,47]) and time-variant reliability analysis concepts are applied, in conjunction with the PBE philosophy, for minimizing the total expected life-cycle cost of civil engineering structures.

The remainder of the paper is organized as follows: In Section 2 the statistical linearization technique is presented/reviewed for MDOF systems. Time-variant reliability concepts are described in Section 3. The performance-based engineering approach is presented in Section 4. The optimization problem formulation is presented in Section 5. An application example involving an RC building subject to wind excitation is presented in Section 6. The paper finishes with some concluding remarks in Section 7.

2. Statistical linearization based response of nonlinear mdof systems

The stochastic differential equation governing the response of a d-degree of freedom structural system subject to stochastic excitation takes the form

$$\boldsymbol{M}\ddot{\boldsymbol{y}} + \boldsymbol{C}\dot{\boldsymbol{y}} + \boldsymbol{K}\boldsymbol{y} + \boldsymbol{g}(\boldsymbol{y}, \dot{\boldsymbol{y}}) = \boldsymbol{w}(t) \tag{1}$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the mass, damping and stiffness matrices, respectively, $\boldsymbol{w}(t)$ is a stochastic excitation vector, $\boldsymbol{y}$ is the relative displacement (drift) between two consecutive stories, and $\boldsymbol{g}(\boldsymbol{y}, \dot{\boldsymbol{y}})$ is an arbitrary nonlinear vector function modeling various forms of nonlinearity. In this paper, statistical linearization is used to transform Eq. (1) into an equivalent linear system of the form

$$\boldsymbol{M}\ddot{\boldsymbol{y}} + \boldsymbol{C}_{eq}\dot{\boldsymbol{y}} + \boldsymbol{K}_{eq}\boldsymbol{y} = \boldsymbol{w}(t) \tag{2}$$

where $\boldsymbol{C}_{eq}$ and $\boldsymbol{K}_{eq}$ are the equivalent damping and stiffness matrices, whose terms are to be determined. Relying on standard statistical linearization results [33] (see also [34]), the equivalent linear system stiffness and damping elements are given by

$$(K_{eq})_{ij} = (K)_{ij} + E\left[\frac{\partial g_i}{\partial y_j}\right] \text{ and } (C_{eq})_{ij} = (C)_{ij} + E\left[\frac{\partial g_i}{\partial \dot{y}_j}\right], \tag{3}$$

where $E[.]$ is the expectation operator. In the case of linear systems of the form of Eq. (2), the frequency response function matrix $\boldsymbol{H}(\omega)$ becomes $\boldsymbol{H}(\omega) = (-\omega^2\boldsymbol{M}_{eq} + i\omega\boldsymbol{C}_{eq} + \boldsymbol{K}_{eq})^{-1}$, whereas in the case of stationary stochastic excitation the spectral excitation-response (input–output) matrix relationship is given by

$$\boldsymbol{S}_{\boldsymbol{y}}(\omega) = \boldsymbol{H}(\omega)\boldsymbol{S}_{\boldsymbol{w}}(\omega)\boldsymbol{H}^{T*}(\omega) \tag{4}$$

In Eq. (4), $\boldsymbol{S}_{\boldsymbol{w}}(\omega)$ represents the power spectrum matrix of the excitation; $\boldsymbol{S}_{\boldsymbol{y}}(\omega)$ represents the power spectrum matrix of the response; and $\boldsymbol{H}^{T*}$ denotes the transpose of the complex conjugate of the frequency response function matrix. Further, the cross-variances of the response displacement and velocity are given by

$$E[y_i y_j] = \int S_{y_i y_j}(\omega) d\omega \quad E[\dot{y}_i \dot{y}_j] = \int \omega^2 S_{y_i y_j}(\omega) d\omega \tag{5}$$

Clearly, Eqs. 3–5 constitute a nonlinear system of algebraic equations to be solved for the unknowns$(K_{eq})_{ij}$, $(C_{eq})_{ij}$, and $S_{y_i y_j}$ (or, equivalently $E[y_i y_j]$). To this aim, an iterative scheme, commonly utilized in statistical linearization applications (e.g. [33]) is also adopted herein. Initial values are assumed for the unknowns $(K_{eq})_{ij}$, $(C_{eq})_{ij}$. Next, $S_{y_i y_j}$ is evaluated using Eq. (4), and $E[y_i y_j]$ is determined by employing Eq. (5). This value is used in Eq. (3) to calculate the equivalent stiffness and damping elements. The iterative scheme is repeated until convergence is reached.

One of the main approximations involved in the standard implementation of statistical linearization (also adopted herein) relates to the fact that the system response is assumed to be Gaussian. This is true for linear systems. Obviously, this is not the case with Eq. (1) since the system is nonlinear. Nevertheless, note that the output of statistical linearization is first- and second-order statistics (system mean response values and variances). Thus, it can be argued that even in cases where the system response probability density function (PDF) deviates considerably from the Gaussian one, the magnitude of this discrepancy is reduced when referring to system first- and second-order statistics. Overall, statistical linearization has been shown in the literature to exhibit satisfactory accuracy for a wide range of systems of engineering interest (e.g. [33,47]). Elaborating further on the accuracy of statistical linearization, it obviously depends not only on the form of the nonlinear function $\boldsymbol{g}(.)$, but also on the nonlinearity and excitation magnitudes. A comprehensive presentation of statistical linearization theoretical as well as accuracy related aspects can be found in Refs. [33,47].

3. Time-variant reliability elements

Using Eq. (5), the time-variant reliability problem for the random system response displacement can be formulated as follows. During a non-zero mean excitation event of specified duration t_E, the response of the oscillator should not exceed the specified limit b_i, where b_i corresponds to some critical response level. The critical response may represent minor damage, gross damage or loss-of-equilibrium, and is further addressed later on in the article. The determination of the above time-dependent probability, known as survival probability, has been a persistent challenge in the field of stochastic dynamics. In this regard, several research efforts have focused on developing versatile MCS based techniques such as importance sampling, subset simulation and line sampling for reliability assessment applications; see [48] and references therein. However, there are cases where the computational cost of these techniques can be prohibitive, especially when large scale complex systems are considered. Thus, there is a need for developing efficient approximate numerical and/or analytical methodologies for addressing the aforementioned challenge (e.g. [49]). One of the first frameworks developed, and adopted herein, is based on knowledge of system response statistics, such as mean out-crossing rates, and usually assumes Poisson distribution based approximations (e.g. [50]).

For a linear system excited by a non-zero mean μ_w Gaussian process, the response is Gaussian and the up-crossing rate can be evaluated as:

$$\nu_y^+(b) = \frac{\sigma_{\dot{y}}}{\sigma_y}\frac{1}{2\pi}\exp\left(-\frac{(b-\mu_y)^2}{2\sigma_y^2}\right) \tag{6}$$

where σ_y and $\sigma_{\dot{y}}$ are given by Eq. (5), by means of some scalar limit state measure (e.g., relative displacement between floors, displacement of top floor, etc.); and μ_y is obtained by taking the expectation of the solution of Eq. (1).

Note that the solution above assumes stationarity of system response. In this regard, many environmental processes acting as structural loading can be described by the arrival of an unknown number of events (winds, storms, sea waves, earthquakes), some of which can be assumed stationary within the specific event duration. For intermittent actions such as wind loading, the crossing rate in Eq. (6) could be further multiplied by the mean rate of occurrence of wind storms. Alternatively, Eq. (6) is computed by adding a turbulent wind model to the annual mean wind speed, as detailed in Section 6.2. Note that for non-stationary excitation cases, such as earthquakes, the current framework can still be applied subject to a rather straightforward generalization. In this regard, the crossing rate in Eq. (6) becomes time-dependent whereas the system response variances become non-stationary. To determine the non-stationary system response variances the standard implementation of statistical linearization delineated in