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Topology optimization of compliant mechanisms with desired structural stiffness

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ABSTRACT

This paper develops a bi-directional evolutionary structural optimization (BESO) method for topological design of compliant mechanisms. The design problem is reformulated as maximizing the flexibility of the compliant mechanism subject to the mean compliance and volume constraints. Based on the finite element analysis, a new BESO algorithm is established for solving such an optimization problem by gradually updating design variables until a convergent solution is obtained. Several 2D and 3D examples are presented to demonstrate the effectiveness of the proposed BESO method. A series of optimized mechanism designs with or without hinge regions are obtained. Numerical results also indicate that the flexibility and hinge-related property of the optimized compliant mechanisms can be controlled by the desired structural stiffness.

1. Introduction

Compliant mechanisms are usually monolithic structures that transfer an input force or displacement to another point through elastic deformation. Different from the rigid-link mechanisms, the motions of compliant mechanisms are derived mainly from the relative flexibility of their components. Such monolithic mechanical devices have numerous virtues such as saving space, reducing fatigue and high stress concentration and without any assembly cost [1]. Therefore, the application of compliant mechanisms has become increasingly prevalent in medical instruments and micro-electro-mechanical systems (MEMS). In spite of various advantages in their application, it is challenging to design a compliant mechanism with desired functions. Generally, there are two main approaches to the design of compliant mechanisms, namely the kinematics-based approach [1,2] and the topology optimization-based approach [3–6].

Topology optimization enables designers to find a suitable structural layout for the required performance. It has attracted considerable attention over the past decades and many different techniques such as the homogenization method [7], Solid Isotropic Material with Penalization (SIMP) method [8–10], level-set method [11,12] and evolutionary structural optimization (ESO) method [13,14] and others [15–17] have been developed. The ESO method is based on a simple concept that inefficient material is gradually removed from the design domain so that the resulting topology evolves toward an optimum. The later version of the ESO method, namely bi-directional ESO (BESO), allows not only to remove elements from the least efficient regions, but also to add elements in the most efficient regions simultaneously [18–20]. It has been demonstrated that the current BESO method is capable of generating reliable and practical topologies for optimization problems with various constraints such as stiffness [21], frequency [22] or energy absorption [23–25].

The design of compliant mechanisms using topology optimization techniques has been exhaustively explored in previous decades [3–6]. Generally, an efficient compliant mechanism as shown in Fig. 1(a) should be flexible enough to produce the expected kinematic motion (flexibility) but should also be stiff enough to resist external forces (stiffness). The flexibility and stiffness characters of a compliant mechanism can be quantified using relationships between the applied forces, the resulting displacements at the input port of the mechanism, and the resulting displacements and reaction forces at the output port of the mechanism. The topology optimization problem has been formulated in a number of alternative ways through utilization of assorted objective and constraint functions due to the inherent multi-objective performance demand. The objective function can be defined by the output displacement, geometric advantage (GA, the ratio of input and output displacements) or mechanical advantage (MA, the ratio of input and output forces) [5,6,26–28] according to the flexibility function of the compliant mechanism. Alternatively, the mutual potential energy (MPE), the strain energy

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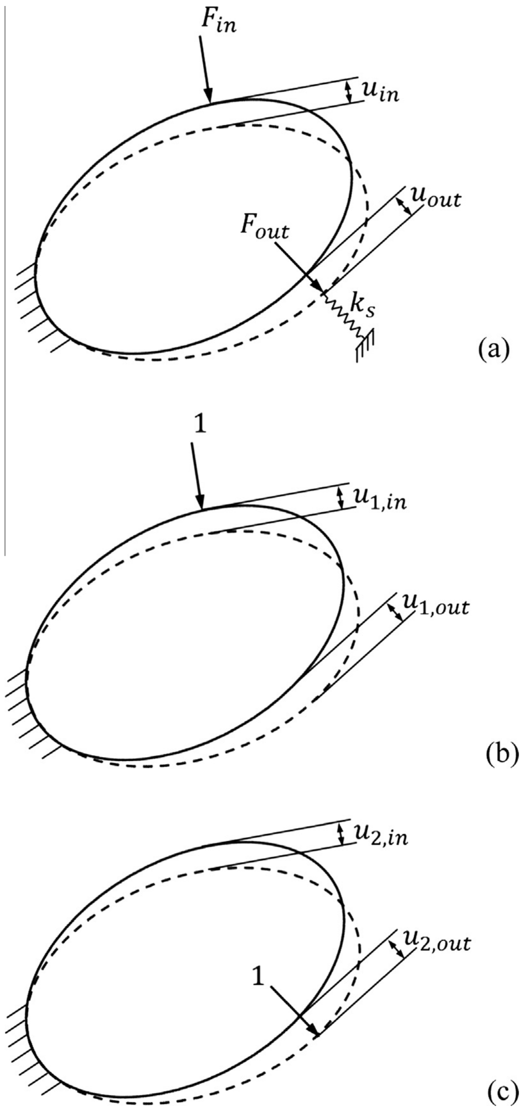

Fig. 1. (a) Compliant mechanism; (b) input unit dummy load case; (c) output unit dummy load case.

(SE) or other equivalent measurements can be used as a single or multiple objective functions to qualify the combination of structural flexibility and stiffness [4,29–33].

Designing compliant mechanisms using topology optimization methods typically results in *de facto* hinge regions in the design models due to the problem formulation. The existence of *de facto* hinge regions makes compliant mechanisms function as rigid-link mechanisms so as to maximize their capability of transferring kinematic motion. Due to the difficulties in manufacturing reliable hinges especially for micro-scale mechanical systems, designing monolithic and hinge-free compliant mechanisms has attracted extensively attention and undergone considerable development in recent years. Rahmatalla and Swan [30] conducted an review on a number of different techniques for eliminating *de facto* hinges in the design of compliant mechanisms. The topology optimization formulation by imbedding wavelet base functions was developed to preclude the formation of *de facto* hinge regions [34,35]. Other approaches attempted to eliminate *de facto* hinge regions include imposing a minimum length constraint [36] or filter schemes [37]. Such morphology-based approaches could greatly reduce the occurrence of one-node connected hinges, but were not entirely effective due to the nature of the optimization problem. Reformulating the problem as a multi-criteria optimization might be an effective way for entirely circumventing *de facto* hinge regions which generally lie along the force path from the mechanism input port to the output port. For example, simultaneously maximizing the flexibility and minimizing the stiffness of the input-restrained structure can achieve hinge-free compliant mechanisms [30,38], and the resulting compliant mechanisms are also stiff and can resist the additional load exerted by the work-piece once it has been secured. Recently, Zhu et al. [39] incorporated this approach to optimize hinge-free compliant mechanisms with multiple outputs. Nevertheless, it should be noted that the stiffness of a compliant mechanism is only equivalent to that of the input-restrained structure when the stiffness of the work-piece tends to infinity.

With a given stiffness of the work-piece, the formation of *de facto* hinge regions must be correlated with the structural stiffness of compliant mechanisms. This paper proposes a new BESO method for optimally designing the flexibility of compliant mechanisms by altering the desired structural stiffness which includes the influence of external loads exerted by the work-piece. The paper is organized as follows: Section 2 reformulates the optimization problem of compliant mechanisms and derives the sensitivities of objective and constraint functions. Section 3 describes the BESO algorithm and its numerical implementation. Section 4 presents numerical examples and discusses the formulation of *de facto* hinge regions. Concluding remarks are made in Section 5.

2. Problem formulation for the design of compliant mechanism

Consider the design domain of a compliant mechanism where F_{in} is the applied force at the input port and u_{out} is the expected output displacement at the output port as shown in Fig. 1a. A spring with a constant stiffness, k_s, is introduced to simulate the interaction between the work-piece and the compliant mechanism. u_{in} is the resulting displacement at the input port and $F_{out} = k_s u_{out}$ is the output force.

When the mechanism behaves in a linear elastic fashion, the displacement field of the mechanism can be calculated according to the displacements caused by the input unit dummy load case (Fig. 1b) and the output unit dummy load case (Fig. 1c). $u_{1,in}$ and $u_{1,out}$ denote the displacements at the input port and the output port of the input unit dummy load case. Similarly, $u_{2,in}$ and $u_{2,out}$ denote the displacements at the input port and the output port of the output unit dummy load case. Thus, the displacements u_{in} and u_{out} at the input and output ports of the mechanism can be found through the superposition of the input unit dummy load case and the output unit dummy load case as

$$u_{in} = F_{in} u_{1,in} - F_{out} u_{2,in} \tag{1}$$

$$u_{out} = F_{in} u_{1,out} - F_{out} u_{2,out} \tag{2}$$

With the relationship of $F_{out} = k_s u_{out}$, u_{in} and u_{out} can be explicitly expressed by

$$u_{in} = F_{in}\left(u_{1,in} - \frac{k_s u_{2,in} u_{1,out}}{1 + k_s u_{2,out}}\right) \tag{3}$$

$$u_{out} = \frac{F_{in} u_{1,out}}{1 + k_s u_{2,out}} \tag{4}$$

Generally, compliant mechanisms should efficiently convert applied force or energy at the input port into desired force or energy at the output port. Within a topology optimization framework, a variety of objective functions are defined to find a compliant mechanism with the desired performance [3–6,21–28]. Basically, the performance of compliant mechanisms can be measured by the characteristics of their flexibility and structural