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A physically-based analytical relationship for practical prediction of leakage in longitudinally cracked pressurized pipes

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ABSTRACT

The present paper focuses on the effect of pressure on the rate of leakage due to crack openings in water distribution system pipes. The aim of the paper is to derive a simple physically-based analytical formula capable of predicting leakage in pipes with longitudinal cracks. Such analytical relationship is derived by properly simplifying a beam model on elastic restraints, recently published by the authors, that predicts the behavior of a longitudinal pipe strip in the cracked zone. The resultant relation is calibrated against three-dimensional finite element simulations and validated through a comparison with several experimental data. The proposed formula can support water providers in the quantification of leakage prediction.

1. Introduction

Water distribution systems worldwide are aging and deteriorating, while the demands on such systems are ever increasing. Particularly, losses from water distribution systems are reaching alarming levels in many towns and cities throughout the world. Water losses comprise various components including physical losses (leaks), illegitimate use, unmetered use, and under-registration of water meters. Leakage makes up a large part, sometimes more than 70% of the total water losses [18,19].

During the last decades, substantial advances have been made in the development of practical methods for understanding and predicting how leakage rates are influenced by the pressure within the distribution system itself [12,17].

The well-known orifice equation, derived from the Torricelli equation [5], describes the flow rate through an orifice with fixed area A [m^2], as follows:

$$Q = C_d A \sqrt{2gH} \tag{1}$$

Q being the water flow [m^2/s], H the pressure head [m], g the acceleration due to gravity [m/s^2], and C_d the discharge coefficient. Such an equation can be generalized to quantify leakage in practice, as follows [17]:

$$Q = CH^{N_1} \tag{2}$$

where C and N_1 are the leakage coefficient and exponent, respectively. Several extensive numerical and experimental studies conducted on the relationship between pressure and leakage in water distribution systems have shown that the rate of flow from a leak is greatly affected by the pressure and that the leakage exponent, N_1, can be larger than the classical 0.5 of the orifice equation [1,10,11,17].

In this context, understanding the failure behavior of pipes and the associated leakage exponents can assist the modeling of the response of a given distribution system to a change in pressure and can greatly improve the management of leakage reduction programmes. Generally, published results on leak behavior report considerations about the leakage exponent, but do not provide effective and practical relationships to estimate leakage, accounting for crack, pipe material, as well as pipe geometry properties. Additionally, detailed material properties, e.g., Young's modulus and Poisson's ratio, are rarely published, and few results have been published on circumferential and spiral cracks.

One possible approach to estimate leakage is phenomenological, that is to calibrate such relationships between water flow and pressure directly on experimental data. However, such an approach would require the availability of several experimental data and the costly calibration of a large number of fitting parameters. Similarly, relationships can be derived by using numerical tools, as finite element analyses. The recent work by Cassa and van Zyl [4] investigates the relationship between pressure and leak area in pipes with longitudinal, circumferential, and spiral cracks by using finite element linear elastic analyses. Sensitivity analyses are conducted to determine the effect of crack, material, and

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geometry properties on pipe behavior and to develop expressions between leak area and pressure head. Such expressions are, however, not validated on experimental data.

Alternatively, starting from the observation that the Torricelli equation does not account for the pipe's deformation in the area of the hole and that leak areas increase as a function of pressure, the approach exploiting the Solid Mechanics theory has been revealed particularly suited in evaluating the influence of the deformability of a cracked pressurized pipe on leakage and in deriving simple practical relationships. As an example, the work by van Zyl and Clayton [18] presents a relationship for pipes with circular holes. Moreover, the first-order shallow shell theory is largely employed to investigate the elastic behavior of a cylindrical shell with a longitudinal crack [13,6]. Another valuable approach is based on the modeling of the cracked pipe as a beam on elastic restraints. In this context, along the line of the earlier work by Hetenyi [14], the paper by de Miranda et al. [7] proposes a simple model based on a beam with continuous elastic restraints. All the cited works, however, do not propose any practical analytical formula between water flow and pressure for leakage prediction.

The present paper aims to support and assist researchers and practitioners in predicting leakage in longitudinal cracked pressurized pipes. To this purpose, the present paper proposes a reliable physically-based formula between water flow and pressure head, accounting for pipe material as well as pipe geometry properties. Following the Solid Mechanics-based approach mentioned above, we obtain such an expression by properly reducing the beam model developed by de Miranda et al. [7] in order to derive a simple analytical expression for the leak area, that accounts for the pipe deformation due to the longitudinal crack. A sole two-parameters correction coefficient is used to calibrate the formula.

The basic idea to derive the formula for the leak area is to consider the longitudinal strip in the cracked part of the pipe as a classical beam with elastic restraints in the tangential plane, without any coupling with transversal displacement. The stiffness coefficient of the elastic restraints is taken as equal to the one of an open annular ring [7] with a two-parameters correction coefficient taking into account for the actual stiffness of the restraints as well as for the other simplifying assumptions. Since studies available from the literature [4,2,9], have confirmed that leak areas expand linearly with pressure under linear elastic conditions, the presented model is assumed linear elastic and does not account for any mechanism of crack propagation. However, we remark that such an assumption may no longer be valid for cracks where plastic deformation or hysteresis occurs [16,9]. Several studies have in fact noted that a nonlinear expansion occurs under high-pressure conditions.

The proposed formula between water flow and pressure is finally validated through a comparison with several experimental investigations [1,10,11]. The obtained results are also compared to those resulting from the application of the relationship by Cassa and van Zyl [4]. Predicted results show that the proposed expression obtained for water loss, despite its simplicity, is reliable and can help in real loss management.

The paper is organized as follows. Section 2 describes the derivation of the pressure-leakage relationship and Section 3 presents the calibration process. Then, Section 4 presents the comparison with experimental results. Conclusions are finally given in Section 5.

2. A predictive pressure-leakage relationship

2.1. Beam-based pipe model

The present Section briefly describes the proposed beam-based pipe model, along the line of the recent work by de Miranda et al. [7].

We start by considering a cylinder pipe of radius, R, and thickness, s, that works at an internal pressure, p, and has a crack assumed as an idealized longitudinal through wall crack, as shown in Fig. 1(a). The crack length is $2L$. We model the cracked pipe by considering a longitudinal beam of cross-section $(b \times s)$, corresponding to the hatched part of the pipe of Fig. 1(a).

We describe the kinematics of the beam by introducing the tangential displacement, u, and we take into account the effect of the remaining part of the cylinder on the longitudinal element by considering continuous tangential elastic restraints, as depicted in Fig. 2.

Thus, the resultant equilibrium equation in terms of the tangential displacement, u, can be written as follows (see Fig. 1(a)):

$$\frac{d^4u}{d(z)^4} + 4\alpha^4 u = q, \qquad -L \leqslant z \leqslant L \tag{3}$$

Here $\alpha^4 = ak_{uu}/4EJ_x$ and $q = Q_g/EJ_x$, where $J_x = sb^3/12$ is the moment of inertia of the longitudinal element cross-section with respect to the x-axis and E the Young's modulus.

The generalized loading, Q_g, is evaluated as the equivalent nodal load, i.e., as the opposite of the fixed-end reaction in tangential direction, as follows:

$$Q_g = -pR \tag{4}$$

As it can be observed by the definition of the term α^4 introduced in Eq. (3), the stiffness coefficient is defined by the product of the stiffness, k_{uu}, and a corrective parameter, a. In particular, the stiffness, k_{uu}, is evaluated by considering an opened annular transverse element of unitary width and cross section $(s \times 1)$, highlighted in Fig. 1(b). In accordance with the classical definition of stiffness, k_{uu} is determined by imposing a unitary value for u with the other displacements equal to zero as follows:

$$k_{uu} = \frac{Es^3}{6\pi(1-\nu^2)R^3} \tag{5}$$

ν being the Poisson's coefficient. We remark that the stiffness coefficient, k_{uu}, is computed by neglecting axial deformation, besides

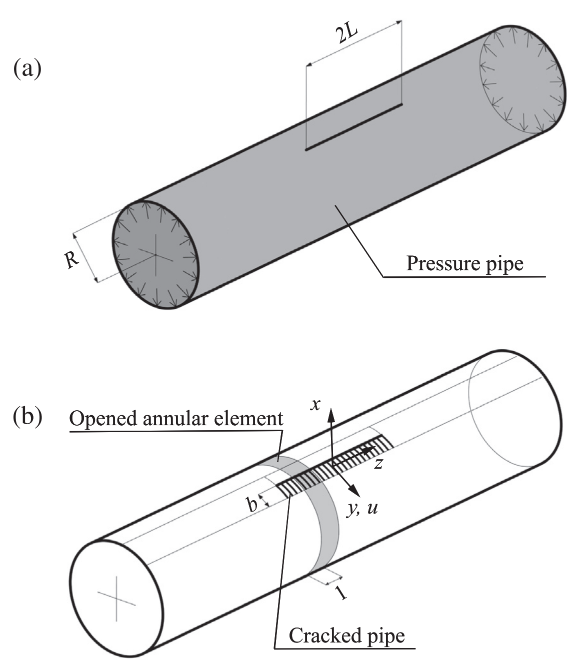

Fig. 1. Longitudinally cracked pressure pipe. (a) Pipe geometry. (b) Opened annular element (gray annular element) and cracked pipe modeled by a longitudinal beam (hatched part).