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Probabilistic model of the crack localization in axially loaded fibrous reinforced concrete bars

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ABSTRACT

This paper proposes a model that explains a phenomenon of only few cracks that significantly widen more than other cracks in tensile reinforced fibrous concrete (RFC) bars. It is a probabilistic model that takes into account both the fibers and conventional reinforcement ratios, where the fiber distribution in the concrete mix is considered random while the conventional reinforcement – deterministic. The model allows the derivation of a binomial probability function, whose maximum value denotes the most expected number of cracks that widen more than the others. It shows a good agreement with available experimental results. A case study demonstrates that for low reinforcement ratios it is most probable that only one or two cracks will significantly widen, and as the conventional reinforcement increases the widths of the cracks become more uniform. This result qualitatively conforms to a similar phenomenon that has been observed in bending experiments.

1. Introduction

Addition of fibers to the concrete mixture increases the material ultimate tensile strain (e.g., [1,2]). Thus, it is expected that the ductility of a structural member, which is made of reinforced fibrous concrete (RFC), will also be enhanced [3,4] and that it will exhibit improved crack control [5–7]. However, there are studies that indicate that RFC structural members exhibited reduced ductility when they included relatively low amounts of conventional reinforcement. Increased steel strains that were caused by crack localization were observed in axial tension specimens, made of normal and high strength concrete (NSC and HSC, respectively) in tests performed at the National Building Research Institute (NBRI) of Technion [8] and in ultra-high performance fibrous concrete (UHPFC) specimens in test performed by Redaelli and Muttoni [9,10]. These researchers reported that their UHPFC tension specimens exhibited good response in service, characterized by thin and closely spaced cracks. Yet at the ultimate limit state, they reported a brittle behavior characterized by crack localization and a sudden failure. This phenomenon of localization, characterized by one or two dominantly wide cracks was also observed in a recently reported study of tension RFC specimens made of NSC and HSC by Deluce and Vecchio [11].

Similar observations have been reported also in flexure (see Fig. 24 in [12,13]), especially in beam specimens with minimal reinforcement ratios [14] and steel fibers showed distinctly lower ductility compared to similar plain reinforced concrete (RC) specimens, whether they were made of normal strength concrete [15,16] (NSC) or high strength concrete [17] (HSC). This result has been observed in beam specimens with reinforcement ratios that were lower than ~0.5%. It was characterized by relatively low ultimate deflection (compared to that of a similar control, plain RC specimens) and by one or two flexural cracks that widened significantly more than the other cracks (unlike the nearly uniform widening of flexural cracks in conventional RC beams). These phenomena diminished at higher reinforcement ratios [15-17].

The cracking localization observed in FRC members may be related to the fibers scatter along the structural member, which has been investigated by several researchers (e.g., [18–20]). In fact, Deluce and Vecchio [11] point to this explanation by describing a localization effect caused at "the weakest section". Their explanation depicts well the reason for local yielding of the rebar at the weakest section of a tension bar. Yet, it does not refer to the possible influence of the conventional reinforcement ratio on this effect and on its observed attenuation in beams with medium to large amounts of conventional reinforcement [15–17]. That is, in flexure, it was observed that in the presence of high conventional reinforcement ratios (compared with the fibers volumetric ratio) the relative effect of the fibers was less pronounced. Therefore, in such

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Nomenclature

f	value of cumulative distribution function at the average
L	length of FRC bar
L_{sg}	length of a segment
L_f	length of a fiber
m	number of 'weak segments'
m_w	most expected number of wide cracks
n	number of cracks
$p(\xi)$	probability density function
$P(\xi)$	cumulative distribution function
p_0	maximum value of the probability density function
$P_A(m)$	binomial probability function
Q	non-dimensional parameter (Eq. (14))
T	tension axial force
V_0	total volume of the bar
V_c	volume of fibrous concrete
$V_{c,sg}$	volume of concrete in a segment
$V_{f,sg}$	average fiber contents in a segment
$V'_{f,sg}$	actual fiber contents in a segment
V_s	volume of conventional steel
λ	L_{sg}/L
ξ	random variable (Eq. (9))
ξ_w	critical value of ξ corresponding to a 'weak segment'
ρ	reinforcement ratio of conventional steel
ρ_f	average volumetric ratio of fibers
ρ'_f	actual volumetric ratio of fibers
σ	standard deviation of ξ
σ_0	standard deviation at which $\xi_w = 1$
σ_{mix}	standard deviation of the fibrous concrete mixture

cases the effect of the 'weak section' is expected to be diminished. Conversely, in the case of relative low amount of conventional reinforcement this effect is expected to become more pronounced.

This paper aims to model the existence of 'weak sections' and their number, depending on the relative amounts of fibers and conventional reinforcement, based on a probabilistic approach.

2. The model

Consider a reinforced concrete bar of length L that includes fibers, subjected to a tension force T, which is applied at the ends of the rebars, Fig. 1. At a certain level of T a crack is initiated in the concrete and as T increases the number of cracks increases as well. Finally, when T reaches a level corresponding to the yield strength of the steel, there are n cracks along the specimen. At this point the bar may be divided into n adjacent sub-elements, each of which includes one crack only, Fig. 1. Note that the sum of the sub-element lengths is equal to the bar's total length, L, i.e., an average length of a sub-element is L/n.

The proposed model has been derived under the following assumptions:

1. The reinforcement and the fibers volume ratios are small ($\rho \ll 1.0$, $\rho_f \ll 1.0$).
2. A typical length of a fiber is much less than a quarter of the bar's length.
3. The lengths of the shorter parts embedded in concrete, of fibers bridging a crack, are uniformly distributed between 0 and half of the fiber length. Therefore, the cumulative function of this distribution is linear (after [1,21]).
4. The model refers only to the action of fibers pullout.

In fibrous concrete, after a crack is formed, it is bridged by the fibers. Therefore, there is a typical part of the bar around the crack,

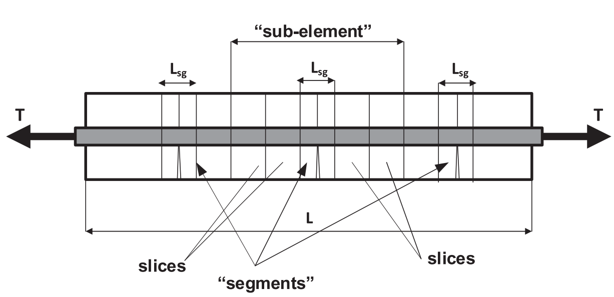

Fig. 1. Problem definition.

which influences this bridging action, that is, where there is slip between concrete and fibers (similarly to the concept of "discontinuity area" in RC [2]). Because this part cannot include more than one crack, its length cannot exceed the length of a sub-element L/n, in which it is included. Therefore, in general, a sub-element may be considered as a set of "slices", where only one of them includes a crack. In the following this cracked "slice" is denoted as a "segment" with length L_{sg}, Fig. 1. The length of this "segment" may therefore be evaluated according to the typical length of the fibers. A "segment" that includes a crack where the bridging action of the fibers is relatively weak will significantly widen at the ultimate state ("weak section" [11]). Such segment is denoted in this work 'weak segment'.

The number of cracks when the steel yields (n) may be obtained experimentally or from calculations of the distance between cracks, which are based on existing analytical (e.g., [22]) or numerical [23] approaches.

The total volumes of the bar, the fibrous concrete and the conventional steel are denoted V_0, V_c and V_s (respectively), such that:

$$V_0 = V_s + V_c \quad (1)$$

The reinforcement ratios of the steel (ρ) and fibers (ρ_f) are defined as follows:

$$\rho = \frac{V_s}{V_0};\ \rho_f = \frac{V_f}{V_c} \quad (2)$$

where V_f is the total volume of the fibers. Note that in these definitions the reinforcement ratio refers to the total volume V_0 whereas the fibers ratio refers to the fibrous concrete volume V_c. Substitution of Eq. (2) into (1) yields:

$$V_c = V_0 \cdot (1 - \rho) \quad (3)$$

According the above definition of a "segment", the volume of concrete and **average** fiber contents in a "segment" are given, respectively, by the following expressions:

$$V_{c,sg} = \lambda \cdot V_c \quad V_{f,sg} = \rho_f \lambda \cdot V_c \quad (4)$$

$$\lambda = L_{sg}/L < \mathrm{MIN}\{L_f/L, 1/n\} \quad (5)$$

where L is the total length of the bar and L_f is the length of a fiber (or a fiber length projection in a direction normal to the crack). The second condition in Eq. (5) ($1/n$) implies that the length of a "segment" cannot exceed L/n (as noted above). The current analysis refers to a state of cracking where the number of cracks n is larger than 2. Accordingly, from Eq. (5) it is apparent that

$$\lambda = L_{sg}/L < 1/2 \quad (6)$$