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Stress–strain–temperature relation for cyclically-damaged structural mild steel

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ABSTRACT

Experimental results suggest that the mechanical properties of mild steel at elevated temperatures are affected by the cyclic load history previously applied to the material. This has great implications when it comes to post-earthquake fire analyzes. Therefore, it is desirable to establish the relationship for each mechanical property, not only as a function of temperature but also the damage induced by the cyclic load history. To achieve this goal, a special class of functions known as Bézier curves have been utilized in this research. These functions are used for the construction of stress–strain curves that depend on temperature and the amplitude of the previously applied strain cycles. Actual experimental results are used throughout the process for calibration and validation purposes. The proposed model proves to be highly versatile in the sense that it can successfully take the effect of temperature and pre-induced strain cycles into account, making it applicable to post-earthquake fire analyzes.

1. Introduction

As a result of the importance of fire resistant structural design, the behavior of different types of steel at elevated temperatures has been the subject of interest for many researchers [1–6]. While one side of the story is extracting behavioral patterns through experiments, the other side is expressing the relationship between stress, strain and temperature. These relationships are developed with the intent of being used either for research or in design guidelines.

Among these relationships, the one given by Ramberg and Osgood [7] is widely accepted. This expression produces a continuous curve commonly used for representing stress–strain responses that do not have distinct yield points. Another continuous curve used for the stress–strain relationship is the one developed by Richard and Abbott [8], which has the capability of tracing the strain softening portion of the response. Both equations given by Ramberg and Osgood [7] and Richard and Abbott [8] involve three parameters; namely the elastic modulus (E), a reference stress (σ_o) and a non-linearity parameter (n) which determines the curvature of the transition between the elastic and plastic parts of the curve.

With the Ramberg–Osgood relation becoming increasingly inaccurate at higher stress levels [9], many modifications have been made to the original equation for its improvement. For example, the advantage of a modified model to trace the stress–strain response of stainless steel, over the original model, is demonstrated in [9]. At the cost of increased complexity, a versatile stress–strain relationship has been presented by Poh [10]. This relation is capable of producing all tangential discontinuities of a complete stress–strain response, including the upper yield point, lower yield point, yield plateau and the initiation of strain hardening. However, the expression requires 10 parameters to trace these features.

As temperature rises, mechanical properties change, even to the point that some parts of the original stress–strain curve vanish, e.g. the upper yield peak and the plastic yield plateau. This calls for a stress–strain–temperature relationship capable of making the transition from ambient temperature to elevated temperatures. To tackle this problem, the expressions given by Rasmussen [9] and Mirambell and Real [11], were used by Chen and Young [12,13] as the basis of a new equation that is valid up to the ultimate stress. Moreover, the stress–strain relation proposed by Poh [10] was effectively utilized in a subsequent paper [14] to include the effect of temperature on the stress–strain response, hence, producing a stress–strain–temperature relation. Kodur et al. [3] compared Poh's relation [14], along with other high-temperature relationships given by American and European standards, to

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available experimental data. They also compared the overall fire resistance predicted by different models.

As a next step, if stress–strain–temperature relations are extended further to include the effect of previously applied loads, such an equation can be used to analyze the response of pre-damaged steel structures at elevated temperatures. This can be directly related to a post-earthquake fire scenario, where a cyclically-damaged structure is exposed to a subsequent fire. The experimental results recently presented by Sinaie et al. [15] cover the changes that the mechanical properties of cyclically-damaged structural mild steel exhibit at elevated temperatures. The present paper aims to establish a relationship for those mechanical properties as a function of temperature and the amplitude of the pre-applied strain cycles. The main purpose of this relation is to act as a user-defined material model for finite element packages such as OpenSees [16] and ABAQUS [17] in the analysis of seismically-damaged steel structures under fire. Moreover, such relations can also be used in semi-analytical formulations developed for the analysis of steel structures at elevated temperatures [18–20].

It is worth mentioning that the behavior of the material under cyclic loading is not the subject of simulation in this paper. Consequently, the effect of the history of cyclic loading is implemented in the proposed model through a parametric value representing the amplitude of the strain cycles. Simulating the cyclic behavior of steel requires more advanced models such as the ones based on the constitutive theory of plasticity [21–26]. Due to the importance of the cyclic response of steel in seismic analysis, its numerical simulation has been explicitly dealt with in another paper by the authors [27].

2. Experimental background

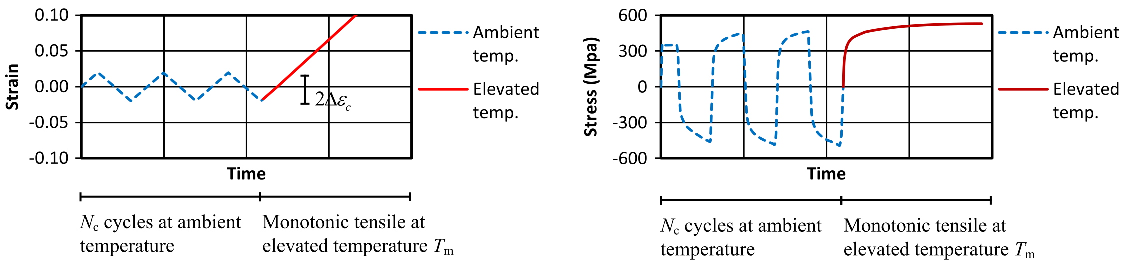
This section provides a brief description of the experiments carried out by Sinaie et al. [15]. Although only relevant information are presented here, details can be found in the original paper. The experiments involved grade 300 mild steel samples, all of which were subjected to a two-phase load history. The first phase was the damage-induction phase in the form of cyclic loading at ambient temperature. This was followed by the second phase where the remaining strength of the pre-damaged material was evaluated through tensile testing at different temperature levels. The complete strain-controlled loading history is illustrated in Fig. 1a as a function of time, whereby $\Delta\epsilon_c$, N_c and T_m act as the test variables denoting the amplitude of the cycles, the number of cycles and the temperature, respectively. The outcome of this loading history is the variation of stress with time given in Fig. 1b. Note that in these figures, the dashed line represents the ambient-temperature cyclic phase of the load history (damage induction), while the solid line represents the elevated temperature monotonic tensile phase (strength evaluation). With all of the samples being of the same material and dimensions, the difference between test cases is in their loading histories. Hence, different test cases are denoted in the form of A□C□_T□, where the blank box (□) in front of A, C and T are respectively filled in by the strain amplitude (in %) of the first phase, the number of cycles of the first phase and the temperature (in °C) of the second phase. Fig. 2 illustrates the stress–strain response during the second phase of the load history for a number of test cases. These figures contain the mechanical properties of cyclically-damaged grade 300 mild steel at elevated temperatures. Numerical reproduction of these variations is the goal of the present paper.

It should be mentioned that the variables of the cyclic phase, i.e. N_c and $\Delta\epsilon_c$ in Fig. 1 have distinct damaging effects on the material. However, for the ranges of N_c and $\Delta\epsilon_c$ covered in the experiments, the damage caused by the amplitude is more prominent than the number of cycles [15]. Therefore, from this point forward, the effect of N_c is omitted and the level of damage is implied through the strain amplitude of the cyclic phase ($\Delta\epsilon_c$). Although this omission is not necessarily valid for $N_c < 3$, it does not harm the goal of this research, since it has been shown that typical earthquakes have at least 3 effective cycles [28]. However, further tests have to be conducted at higher number of cycles to reach a definite conclusion for values outside the range considered in this research.

It should also be mentioned that in the two-phase experiments conducted by Sinaie et al. [15], the elevated-temperature phase followed the cyclic damage-induction phase within a time gap small enough to not allow for significant strain aging. Hence, when it comes to the analysis of cyclically-damaged steel structures under fire, using the experimental results of [15] is limited to scenarios where the fire immediately follows the seismic loading. The strain aging effect as well as the post-cooling behavior of the steel material is currently being investigated at Monash University. It is worth noting that the generality of the formulations given in the following sections allows such effects to be easily implemented in the analysis, either by a single overall parameter, or as independent parameters.

3. Current stress–strain–temperature relations

In the course of expressing stress as a function of strain and temperature, various explicit relations have been presented by different researchers. Examples of such equations are described and compared to each other by Kodur et al. [3] and Poh [14]. Due to the flexibility and robustness of the equations proposed by Poh [14], a modified version of it is used in this paper for comparative reasons. The original relation involving 10 parameters ($\beta_1 - \beta_{10}$) is hereby modified by setting $\beta_6 = 0$. Doing so simplifies the equation by eliminating the upper yield peak from the original model. Note that by setting $\beta_6 = 0, \beta_7$ also vanishes from the equation. In order to be consistent with their original definitions, the remaining β_i's are not re-indexed here. Hence



(a) (b)

Fig. 1. Time variation for the multi-phase loading history (a) strain variations and (b) stress variation.