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Theoretical response of a simply supported beam with a strain rate dependant modulus to a moving load

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ABSTRACT

Moving load problems typically consider a structural material with properties that do not vary while the load traverses the structure. However, there is evidence that for some materials the structure will respond with a higher modulus of elasticity than that corresponding to a static test for sufficiently high strain rates. This paper investigates the variation in strain rate of a simply supported beam made of a viscoelastic material traversed by a moving load and its effect on the modulus of elasticity. The influence of speed and magnitude of the moving load on the displacement and strain responses of the beam is discussed.

1. Introduction

Large dynamic loads that are applied very rapidly, such as impact loading, have been shown to largely influence the way a structural material behaves [1–6]. Modulus of elasticity, and tensile and compressive strengths are some of the material properties affected by the dynamic nature of the applied load. This phenomena is commonly found in construction materials such as asphalt [7] or concrete [1,5], which loaded at constant strain rate, exhibit significant increases in the modulus of elasticity as strain rate is raised. In the case of reinforced concrete tests, the increase in modulus of elasticity with strain rate strongly depends on the method of testing [3,8]. No significant increase is identified for steel [8–10]. In addition to impact studies, differences between the 'static' modulus of elasticity, E_s, (i.e., derived from static loading tests) and the 'dynamic' modulus, E_d, are also noticeable when the latter is derived from measured frequencies of vibration [11]. Different analytical techniques (eigenvalue analysis, Rayleigh energy method) are employed to derive a relationship between frequency and E_d [12]. Finally, simple equations such as "$E_s = 0.83E_d$" and "$E_s = 1.25E_d - 19$ (both E_s and E_d in GPa)" are proposed for concrete by Lydon and Balendran [13] and BSI [14] respectively [15].

It is then acknowledged that high strains can lead to a 'dynamic' modulus significantly different from the 'static' modulus. Two questions arise here: (i) "what's the static strain rate threshold beyond which the material will start to develop viscoelastic properties?" and (ii) "how E_d relates to E_s at high strain rates?". In relation to the first question, Bischoff and Perry [2] consider that strain rates can vary from $10^{-6}\,s^{-1}$, for the case of a static load application, to $10^{3}\,s^{-1}$ for hard impact or an explosion. For loads moving across a structure, the strain rate will depend on the mechanical properties of the structure, the magnitude and speed of the load, but it can be assumed to be somewhere between the static ($>10^{-6}\,s^{-1}$) and that associated to earthquake loading ($<10^{-2}\,s^{-1}$). The response of a beam to a moving load has been investigated for numerous scenarios in the literature: uniform and tapered sections [16], straight and curved alignments [17], simple supported and continuous spans [18], un-cracked and cracked sections [19,20], Euler–Bernoulli and Timoshenko type [21], etc. A thorough review can be found in [22–26]. However, these moving load investigations use material properties that do not vary during the load crossing; in particular, the influence of strain rate on the behavior of the material is neglected. While the latter will hold true for some materials or applications of small loads at low speeds, recent research confirms the impact of high strain rate on the properties of concrete in bridges [1,2,27]. Further evidence can be found in the Bridge Weigh-In-Motion literature, where the static moment in a bridge is related to the measured strain (prior removal of dynamics and noise via a low-pass filter) by scaling the influence line using a calibration factor. The calibration factor is representative of the section modulus and modulus of elasticity at the measurement point and it is obtained by driving a vehicle(s) of known configuration with different speeds and loading conditions over the bridge. However, some

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Bridge Weigh-In-Motion sites have revealed a tendency of the factor to increase with higher loads and speeds [28]. These sites suggest that the mechanical properties of the bridge are affected by high strain rates and are one of the drivers for the theoretical investigations in this paper. By the first time in the literature, consideration is given to how the strain rate and modulus of elasticity of a beam made of a viscoelastic material change over time due to a moving load. Given that E_d is higher than E_s, as strain rate rises, the structure will behave in a stiffer way and react to the applied load with a smaller response than initially expected. Simulations are carried out for simply supported beams with different mechanical properties. Load speeds and load magnitudes are also varied in order to assess the impact of introducing a viscoelastic material in the moving load problem.

2. Model to simulate the response of a beam to a moving load

The moving load is represented by a constant force and the underlying structure is modelled as a simply supported discretized finite element Euler–Bernoulli beam of constant rectangular cross-section as sketched in Fig. 1.

Although this simplistic model assumes that the mass of the moving load is much smaller than that of the bridge (i.e., the interaction between both is neglected), it is still widely used in research and in practice. For instance, it has resembled patterns of dynamic amplification versus speed measured in bridges [20]. Therefore, it is deemed to be sufficient for the aim of evaluating the strain rate that may develop in the beam and its potential effect on the overall response. Details on its implementation [29] are provided here.

Two degrees of freedom per beam node are considered in this model (vertical displacement u_j and rotation θ_j for each elementary beam j as shown in Fig. 2). Therefore, the elementary stiffness matrix $[K_j]$ relating forces and moments to these degrees of freedom at each individual discretized beam j is given by:

$$[K_j] = \begin{bmatrix} \frac{12E_jI_j}{L_j^3} & \frac{6E_jI_j}{L_j^2} & -\frac{12E_jI_j}{L_j^3} & \frac{6E_jI_j}{L_j^2} \\ \frac{6E_jI_j}{L_j^2} & \frac{4E_jI_j}{L_j} & \frac{6E_jI_j}{L_j^2} & \frac{2E_jI_j}{L_j} \\ -\frac{12E_jI_j}{L_j^3} & -\frac{6E_jI_j}{L_j^2} & \frac{12E_jI_j}{L_j^3} & -\frac{6E_jI_j}{L_j^2} \\ \frac{6E_jI_j}{L_j^2} & \frac{2E_jI_j}{L_j} & -\frac{6E_jI_j}{L_j^2} & \frac{4E_jI_j}{L_j} \end{bmatrix} \quad (1)$$

where E_j, L_j, and I_j are the modulus of elasticity, length, and second moment of area of each elementary beam. These elementary stiffness matrixes are assembled into the global stiffness matrix $[K]$ (Fig. 1).

Initially, the modulus of elasticity E_j is adopted to be the 'static' modulus E_s. However, in the case of using a strain rate dependant material, the modulus of elasticity E_j (and hence, the stiffness E_jI_j)

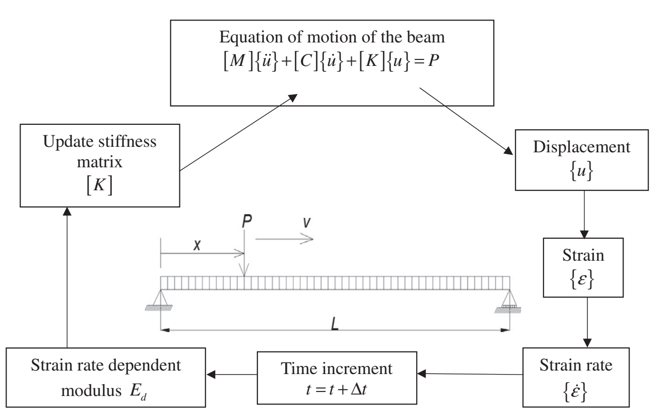

Fig. 1. Simulation model.

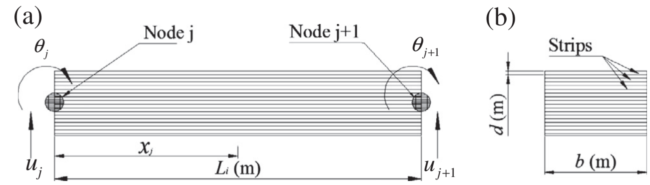

Fig. 2. Elementary beam element: (a) elevation and (b) cross-section.

may adopt a value of 'dynamic' modulus E_d that can vary at each point in time. These changes in stiffness are updated using an equivalent moment of inertia as follows. The cross-section of each elementary beam is discretised into strips as in Fig. 2. In this figure, d is the depth of each strip, which has been adopted to be 0.003 m for the simulations in this paper (i.e., 200 strips for a total element depth of 0.60 m).

The displacements of the beam at each node are calculated using the equation of motion in Fig. 1 and the strain $\varepsilon_{j,k}$ is estimated using Eq. (2) for each kth strip of beam element j at each time step:

$$\varepsilon_{j,k} = \frac{-y_k}{L_j^3}\left\{(6L_j - 12x_e)\left(4L_j^2 - 6L_jx_e\right)(-6L_j + 12x_e)\left(2L_j^2 - 6L_jx_e\right)\right\}\begin{Bmatrix} u_j \\ \theta_j \\ u_{j+1} \\ \theta_{j+1} \end{Bmatrix} \quad (2)$$

where y_k is the distance from the center of the kth strip to the neutral axis of the entire cross-section and x_e is the distance where strain is obtained at the beam element. The value $x_e = L_j/2$, where L_j is the length of the beam element, is used in the simulations to calculate an average strain for each element. Strains from Eq. (2) are then used to derive the strain rate $\dot{\varepsilon}_{j,k}$ for each element j and strip k (Eq. (3)):

$$\dot{\varepsilon}_{j,k}(t) = \left(\frac{\varepsilon_{j,k}(t) - \varepsilon_{j,k}(t - \Delta t)}{\Delta t}\right) \quad (3)$$

where Δt is the time increment. This strain rate is used to calculate the 'dynamic' modulus E_d corresponding to each strip k using Eq. (4) which relates E_d to E_s and $\dot{\varepsilon}$:

$$\begin{cases} E_d(t) = E_s\left(\frac{\dot{\varepsilon}_{j,k}(t)}{\dot{\varepsilon}_0}\right)^{\gamma} \text{ for } \dot{\varepsilon}_k > \dot{\varepsilon}_0 \\ E_d = E_s \text{ for } \dot{\varepsilon}_k \leqslant \dot{\varepsilon}_0 \end{cases} \quad (4)$$

Here $\dot{\varepsilon}_0$ is 'static' strain rate threshold (or value in s^{-1} above which the modulus becomes strain rate dependant) and γ is an empirical constant. In Eq. (4), E_s is the 'static' modulus of elasticity (N/m^2). This equation is a typical constitutive model for viscoelastic materials, which has been adopted by the Comité Euro-International du Béton (CEB) Model Code. Values of γ of 0.026 and $\dot{\varepsilon}_0$ of $3 \times 10^{-6}\,s^{-1}$ and $30 \times 10^{-6}\,s^{-1}$ in tension and compression respectively are recommended for concrete [30], although $\dot{\varepsilon}_0$ can vary between 1×10^{-6} and $60 \times 10^{-6}\,s^{-1}$ in the literature [1,6,10]. For asphalt, it has been found that the modulus of elasticity is most sensitive to dynamic loads for $\dot{\varepsilon}_0 \geqslant 1.7 \times 10^{-5}\,s^{-1}$ [31]. For steel, experimental investigations have shown that the modulus of elasticity remains unchanged at high strain rates [10,32] but the strain rate has an impact on the steel's yield strength, ultimate tensile strength and strain. In the case of concrete, Yon et al. [27] use γ values of 0.065 and higher. They assume a 'static' strain rate threshold $\dot{\varepsilon}_0$ of $2.5 \times 10^{-3}\,s^{-1}$ based on experimental results, and for a maximum strain rate of $0.24\,s^{-1}$, they find an increase in the compressive and tensile moduli of concrete of 41% and 60%, respectively. Most of the literature on calibrating γ values for different materials uses experimental data conducted on a cubic specimen under impact loading to create high levels of strain rate at a single point.