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# A morphogenesis method for shape optimization of framed structures subject to spatial constraints

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ABSTRACT

A morphogenesis method is proposed for the topology and shape optimization of framed structures subject to spatial constraints. This combines direct elemental addition, or elimination, and free nodal shift, or restricted nodal shift related to the structures geometry. The optimization is based on elemental and nodal sensitivity information to generate or amend the structural topology and adjust the nodal positions to achieve a structure with minimum strain energy. In this method, the design parameters, such as supporting conditions, spatial constraints, etc, have significant influence on the final structural form; so various structural forms can be obtained by changing these design parameters in the project design phase. Several numerical examples are provided to illustrate the validity of this method and the mechanical behaviour of these structures. Results show that this can effectively reduce the structural bending moments and ensure sufficient structural stiffness.



## 1. Introduction

The selection of structural shape affects structural characteristics, and architectural functions and visual appearance. There is a close relationship between architectural design and structural design, because they influence and restrict each other. At present, architects often propose conceptual designs from the point of view of architectural functions and visual effects according to their experience and theories, then verify and correct them. In the design process, structural analysis is just a tool for realizing an architectural intention, but this ignores the positive impact of structural rationality on the architectural intention. Structural morphogenesis is a new subject, which seeks to generate better shapes by combining architectural intention and structural rationality, which is significant in the architectural design stage. The structural morphogenesis method is based on optimization theories, especially shape and topology optimization. Beghini et al. [1] also proposed a similar viewpoint that connecting architecture and engineering through structural topology optimization. They discussed the importance of a close collaboration between these disciplines, and offered an alternative approach to generate new, integrated design ideas by means of a tailored structural topology optimization framework, which can potentially be of benefit to both the architectural and structural engineering communities.

Framed structures are widely used in civil engineering. Many researchers have focused on the structural morphogenesis of framed structures, and proposed many correlative theories and methods. In the early stage, Dorn et al. [2] proposed the ground structure method (GSM). In this method, the cross-sectional areas were considered as continuous design variables and members with vanishing cross-sectional areas were removed to obtain optimal topology. The basic idea of the GSM was widely used in truss topology optimization. For example, Rule [3] proposed an optimized growth for automatic truss design. Bojczuk [4] studied the optimal topology and configuration design of trusses with stress and buckling constraints. Ohsaki [5] and Burns [6] presented a simultaneous optimization method for topology and geometry of a regular plane truss, and he considered the nodal cost for structural topology. Wang et al. [7] developed a sensitivity analysis method based on nodal coordinates with multiple displacement constraints. Stromberg et al. [8] described an integrated topology optimization technique, with concurrent use of both continuum four-node quadrilateral finite elements and discrete two-node beam elements, to design structural braced frames for high-rise buildings. Kaveh et al. [9] adopted Ant Colony Optimization and finite element analysis in topology optimization of structural models to find the stiffest structure with a certain amount of material, based on the element's contribution to the strain energy. Almeida et al.

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[10] employed Smooth Evolutionary Structural Optimization to carry on comparative analysis of strut-and-tie models. Kawamura et al. [11] investigated the problem of shape topology optimization for planar truss and single-layer lattice structures using a genetic algorithm (GA). Bendsøe [12] studied topology optimization of trusses by taking the nodal positions as design variables. Martínez et al. [13] put forward a growth method for the optimal design in a sequential manner of size, geometry, and topology of plane trusses without the need of a ground structure. Hagishita and Ohsaki [14] developed the ground structure method and put forward the growing ground structure method with element addition. McKeown [15] developed growing optimisation for pin-jointed structures which introduced joints, either one by one or in symmetric groups, and used a deflection-variable method to simultaneously optimize geometry and layout. Luh and Lin [16] employed a two-stage particle swarm optimization for the optimal design of truss-structures. Although all these methods are still in the exploratory stage, they offer the possibility of determining structural shape in design by theoretical methods.

There are three key problems with these methods. First, trusses have mainly been studied, so using cross-sectional area as a design variable has been convenient. However, for framed structures, cross-sectional properties have a complex relationship with structural stiffness. The relationship between cross-sectional area and moment of inertia is not independent and is determined by the cross-sectional form of the members. Hence, it is unreasonable to treat the cross-sectional area as a design variable. Therefore, these methods are not suitable for the topology optimization of framed structures. Since the cross-sections of members are not allowed to have an arbitrary form, it is practical to develop an optimization method to create optimal framed structures consisting of members with the same cross-section. In addition, members with the same cross-section will also reduce the construction costs due to convenience and efficiency of component manufacture. Second, these methods did not consider the spatial constraints in the architectural design, so the structures achieved by these methods can rarely be applied in practical situations. Third, the quality of the solution depends on the locations of the nodes and the connectivity of the bars of the initial ground structure [14]. The way that the topology optimization is achieved by size optimization in GSM is not very efficient for structural morphogenesis in design.

Continuum optimization methods provide suggestions for the improvement of topology optimization for framed structures. Xie et al. [17–21] proposed the ESO (the abbreviation of Evolutionary Structural Optimization) method and the BESO method, which are effective approaches for topology optimization. The basic idea is to eliminate inefficient materials and gradually adding materials near highly efficient materials to make the structure evolve into a rational one. Cui et al. [22] put forward computational morphogenesis of 3D structures by the extended ESO Method and applied it to practical engineering projects. It is wise to learn from the BESO or Extended ESO methods and this paper develops a method of direct beam element addition and elimination for the topology optimization of framed structures. In addition, the core idea of the constructal theory [23–25] indicates that various shapes and the structure of matter in nature are generated from the tendency to obtain optimal performance, which also provides enlightenment as to how to construct efficient structures. In this paper, minimizing strain energy produced by the same loads is taken as the principle for generation of an efficient structure.

Structural morphogenesis should consider three requirements: structural mechanical performance, visual effects and spatial requirements from architectural functions. The structural strain energy produced by external loads may be taken as the index for evaluating structural rationality, because the smaller the strain energy, the bigger the structural stiffness. In this paper, the design variables are the elements and nodal coordinates; the cross-section is the same for all of the structural members. In framed structures, the sensitivity number of the elements, or nodes, denotes the contribution of bearing loads of an element or the changing degree of strain energy corresponding to the change of the nodal position.

The elemental efficiency, in respect of the bearing loads, is a measure of the distribution of elements or structural weight during the minimization of strain energy for elemental addition/elimination. A morphogenesis method for framed structures is proposed, adopting the crossover operation of direct elemental addition/elimination and nodal adjustment according to the sensitivity characteristics of elements and nodes and the relationship between them and the strain energy. The basic idea is to remove inefficient elements and add new elements near highly efficient elements in combination with nodal shifts. According to the properties of the strain energy sensitivity, the nodal shift can be seen as a process of structural self-improvement and the elemental addition/elimination as an exchange process of energy and materials to evolve to an efficient structure with optimal performance. This combination can improve structural efficiency. Obviously, this is more effective than GSM for topology optimization. In addition, the method considers spatial constraints to ensure the requirements of the architectural functions when nodal sensitivity numbers are calculated. The method is coded in the FORTRAN language.

The final structural shape is closely related to the design parameters, such as initial shape, support conditions, spatial conditions, etc., therefore, a variety of reasonable shapes can be obtained by adjustment of the design variables according to visual requirements. The effectiveness of the method is verified in section 4 using numerical examples.

## 2. The morphogenesis method for a framed structure

This section presents the sensitivity characteristics of the strain energy of the elements and nodes and the concept of the crossover operation for direct elemental addition/elimination and nodal adjustment considering the spatial constraints.

### 2.1. The morphogenesis problem for a framed structure

The morphogenesis problem for structural stiffness maximization subject to spatial constraints can be formulized as follows:

$$\begin{cases} C(\boldsymbol{P}) \rightarrow minimum \\ s.t. \quad S \subset \Omega_0 \\ \qquad \sigma_{max} \leqslant \sigma_0 \\ \qquad \delta_{max} \leqslant \delta_0 \end{cases} \tag{1}$$

where $C$ is the structural strain energy; $S$ denotes structural shape; $\Omega_0$ is the design allowable space including spatial constraints which can be expressed by a B-spline curve/surface or a surface/curve analytical equations; $\sigma_{max}$ is the maximum stress and $\delta_{max}$ is the maximum displacement. $\boldsymbol{P}$ is the design variable, such as nodal coordinates and elements (Unless otherwise noted, the elements are beam elements, and the nodes are rigid-joints). The minimization of strain energy means maximization of structural stiffness. Structures derived by this method always tend to evolve into structures with higher stiffness and uniform stress distribution. Therefore, the constraints of stress and displacement will usually be satisfied with an increase in structural stiffness. If not, the cross section of members should be increased to satisfy these constraints of stress and displacement.