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Numerical simulation of reinforced concrete beam/column failure considering normal-shear stress interaction

X. Long [*], J.Q. Bao, K.H. Tan, C.K. Lee

School of Civil and Environmental Engineering, Nanyang Technological University, 50 Nanyang Avenue, Singapore 639798, Singapore

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ABSTRACT

Fibre beam elements have vast applications to simulate the behaviour of reinforced concrete beam/column members. If normal-shear interaction is ignored, separated normal and shear constitutive laws are usually used in fibre beam elements, which is invalid to simulate shear failure in beam/column members with small or medium shear span-to-depth ratios. In this paper, a fibre beam element is treated as a degenerated solid element, and a unified concrete constitutive model is proposed for the degenerated solid element. Different from the separated constitutive models, the unified concrete constitutive model incorporates normal and shear behaviour based on three dimensional stress–strain states and, thus, normal-shear interaction is naturally considered. Moreover, compressive failure, shear failure and tensile failure are accounted for. Therefore, beam/column members with a wide range of shear span-to-depth ratios can be simulated with the degenerated solid element considering normal-shear interaction. Lastly, a variety of examples are presented to demonstrate the applicability and reliability of the proposed method by comparing numerical predictions with experimental results.

1. Introduction

When simulating the behaviour of reinforced concrete (RC) beam/column members, a three dimensional (3D) fibre beam element formulation is often preferred in order to obtain satisfactory accuracy with acceptable computational cost [1,2]. Compared with conventional beam elements, more detailed stress and strain profiles can be achieved and different materials can be realised in fibre beam elements by using a fibre model at the cross-sectional level. For example, concrete fibres can be employed to discretize the cross-section of beam/column members while steel fibres with equivalent areas can be assigned at the steel reinforcement locations. In addition, material nonlinearity at the fibre level, such as cracking and crushing of concrete fibres and yielding and fracturing of steel fibres, can be conveniently taken into account based on the plane section hypothesis.

At each fibre cross-section, there are one normal stress component along the longitudinal (fibre) axis and two orthogonal shear stress components along the strong and the weak axes, respectively. For simplicity, the constitutive laws for the normal stress and shear stresses are independent of each other. Many of the widely accepted and validated concrete constitutive laws published for concrete under uniaxial compression, such as the Modified Kent and Park model [3], the Mander's model [4,5] and some other models [6,7] proposed recently, can be adopted to describe the normal stress–strain relationship in concrete. This type of one-dimensional concrete models is often termed as *uniaxial concrete models*. Both the nonlinear finite element software Engineer's Studio developed by Tokyo University [8] and OpenSees developed by the University of California, Berkeley [9], which are widely used for academic research nowadays, only consider *uniaxial deformation behaviour* of reinforced concrete in their fibre beam elements. By assuming an elastic shear stress–strain relationship, *uniaxial concrete models* are fairly reasonable if flexural failures dominate the failure mode.

For the prediction of shear failures in RC beams, empirical shear models based on experimental studies have been adopted in many previous numerical studies on fibre beam element formulations [10–12]. The shear models proposed by Priesley et al. [13], Sezen and Chowdhury [14] and Filippou et al. [15], and design regulations, such as Eurocode 2 [16], are often employed to describe the shear stress–strain relationship of concrete fibres. However, it should be noted that these constitutive laws for shear stress are independent of normal stress and the interaction between normal and shear stresses in the fibre beam elements is neglected.

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[*] Corresponding author. Tel.: +65 8524 3665; fax: +65 6791 0676.
E-mail addresses: LONG0026@ntu.edu.sg (X. Long), BJQ05@mails.tsinghua.edu.cn (J.Q. Bao), CKHTAN@ntu.edu.sg (K.H. Tan), CCKLEE@ntu.edu.sg (C.K. Lee).

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Nomenclature

c	concrete softening function
$d_{\max}$	maximum aggregate size
$\boldsymbol{D}_6$	tangential material matrix with a dimension of 6×6
$\boldsymbol{D}_3$	3×3 matrix extracted from tangential material matrix $\boldsymbol{D}_6$
e	eccentricity parameter of out-of-roundness
f	failure function
f_c	uniaxial compressive concrete strength
f_t	uniaxial tensile concrete strength
g	potential function
G	intact shear modulus
G^c	cracked shear modulus
G_f	fracture energy
h	crack bandwidth
I_1, J_2, J_3	invariants of the principal stress tensor components
k	concrete hardening function
k_0	value defining the onset of plastic flow
m	friction parameter
t	relevant to the slope of the softening function
r	elliptic function
α_{p0}	brittleness index of concrete, relevant to the ductility and post-peak stress–strain relationship
α'_{p0}	parameter in potential function
$\sigma_1, \sigma_2, \sigma_3$	principal stress tensor components
$\Delta\boldsymbol{\sigma}_6$	incremental stress vector
$\Delta\boldsymbol{\sigma}^{un}$	incremental stress vector of $\Delta\sigma_y$, $\Delta\sigma_z$ and $\Delta\sigma_{yz}$
$\Delta\boldsymbol{\sigma}_3$	incremental stress vector of $\Delta\sigma_x$, $\Delta\sigma_{xy}$ and $\Delta\sigma_{xz}$
$\Delta\boldsymbol{\varepsilon}_6$	incremental strain vector
$\Delta\boldsymbol{\varepsilon}^{un}$	incremental strain vector of $\Delta\varepsilon_y$, $\Delta\varepsilon_z$ and $\Delta\varepsilon_{yz}$
$\Delta\boldsymbol{\varepsilon}_3$	incremental strain vector of $\Delta\varepsilon_x$, $\Delta\varepsilon_{xy}$ and $\Delta\varepsilon_{xz}$
ε_v^p	plastic volumetric strain
ρ	cylindrical coordinates of hydrostatic length in the Haigh–Westergaard coordinates
ξ	deviatoric length
θ	lode angle

Alternatively, in order to consider the interaction between normal and shear stresses in fibre beam elements, Petrangeli et al. [17] proposed a concrete law based on the microplane theory, which assumes that the cross-sectional strain profile is the superposition of the normal strain and the shear strain by imposing equilibrium between the concrete and the transverse steel reinforcement. The couple effect of the normal and shear stresses is taken into account by lateral expansion of the concrete section. However, this model requires a thorough reformulation of the concrete constitutive law and demands a considerable computational cost. Papachristidis et al. [18] presented a forced-based fibre beam element for structural nonlinear analysis with consideration of shear modelling based on a J_2 3D plasticity constitutive law. However, their study is limited to steel structures. Different from steel material, the plasticity-based constitutive laws for concrete are much more complicated in terms of yielding criterion and flow rule. Following the assumptions of the Modified Compression Field Theory, Navarro-Gregori et al. [19] proposed a section model into a 2D beam element formulation by including a variable shear strain profile with a parabolic-straight-line shape for rectangular RC cross-sections under monotonic loading conditions. Nevertheless, this model is limited to 2D cases as the shear strain profiles could be much more complicated in both cross-sectional directions.

In principle, any stress–strain state at any material point is three dimensional, which consists of six components for both stress and strain. Apparently, the ideal way to account for the interaction between normal and shear stresses in concrete fibres is to employ plasticity-based constitutive laws for concrete. In this paper, based on the concept of *degenerated solid element*, the fibre beam element is treated as a special 3D solid element to consider the interaction between the normal and shear behaviour of concrete so that a 3D plasticity-based concrete model can be adopted as the concrete constitutive law. An obvious advantage of this approach is that the stress state in the fibre beam element is determined from a 3D material stress–strain state and, thus, the couple effect of normal and shear stresses in concrete fibres can be fundamentally accounted for by imposing certain constraint equations due to stress simplification resulting from transformation of beam elements from solid elements. In addition to the plasticity model to simulate concrete deformation in compression, a fracture model should also be incorporated to simulate concrete tensile behaviour in RC beam members [20].

Generally, there are two fundamental ways to simulate tensile failure of concrete by differentiating whether cracks are discrete or smeared [21,22]. As for the discrete crack model, cracks are explicitly accounted for as displacement discontinuities. From the point of view of finite element simulations, discontinuities induced by cracks in the displacement field can be simulated as the interface of finite elements by cohesive crack model [23,24] (or named as fictitious crack model) based on the principles of fracture mechanics and energy balance, which is suitable to the studies of initiation and propagation of cracks known *a priori*. However, when a crack is identified to propagate through finite elements, the region has to be re-meshed, which could result in a topological problem that is not straightforward to handle [25]. Instead of remeshing the numerical model, the displacement discontinuities within finite elements can be simulated by extended finite element method (XFEM) [26,27] which enriches the nodal shape function of finite element formulation based on a local partition of unity. As the fundamental idea is close to the cohesive crack model, XFEM has similar challenges to deal with the crack propagation, branching and intersection [28].

On the other hand, when omitting localised cracking simulation, the cracks in the smeared crack model are considered to be spread across the elements by changing their constitutive equations. The smeared crack model is similar to the concept of crack band model which is in good agreement with experimental studies on fracture and size effect and is also convenient to be programmed [29–31]. To reflect tensile softening due to cracking, the stress–strain relationship can be manipulated by failure surfaces such as Rankine criterion combined with certain softening rules [32]. Alternatively, a simple fracture model can be employed with a modified tensile stress–strain relationship, in which the fracture criterion is based on the concrete tensile strength. This type of simple fracture model has been widely used in degenerated solid element implementations [33–36].

Some other concrete models are also proposed in order to combine the merits of the discrete and smeared crack models, such as cohesive segments method [37,38], which exploits the partition-of-unity property of finite element shape functions, and non-local regularization technique [39–41], which takes into account the spatial average of local damage over a characteristic length of micro-structure. However, these models require material parameters difficult to be derived from existing material tests, which hinders their applications to practical finite element analysis [22].

To avoid complicating concrete constitutive relations in the beam element formulation and stabilize convergence when finding the optimum return point on the active failure surfaces [42], the