Engineering Structures 74 (2014) 218–228

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Extended sandwich model for reinforced concrete slabs: Shear strength with transverse reinforcement

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ARTICLE INFO

Article history:
Received 3 February 2013
Revised 2 March 2014
Accepted 19 May 2014
Available online 13 June 2014

Keywords:
Aggregate interlock
Deformation capacity
Finite element modeling
Limit analysis
Reinforced concrete
Reinforcement
Shear strength
Slabs
Stiffness
Stirrups

ABSTRACT

In this paper, the description of the shear strength of orthogonally reinforced concrete slabs with transverse reinforcement by the newly developed extended sandwich model is presented. Based on a sandwich model, the slab element is subdivided into two cover elements and a core element. The applied in-plane forces on the cover elements are treated with the cracked membrane model. Regarding shear transfer, rotating crack faces that are able to transfer shear stresses by aggregate interlock are assumed in the core, whereas the crack orientation relative to the slab plane is determined by the crack pattern of the covers. The introduction of stressed crack faces in the core enables a subdivision of the applied shear force into a concrete and a steel contribution, allowing the determination of the required minimum transverse reinforcement ratio that enforces a ductile flexural failure. A brittle shear failure is eliminated by providing a transverse reinforcement even if it is a minimum transverse reinforcement that is not able to resist the applied shear force by itself. In addition, the extended sandwich model enables a general treatment of the deformation behavior. Verifications against experimental data generally show a good agreement. The influences of a deviation of the principal shear and moment direction from the direction of the in-plane reinforcement as well as the transverse reinforcement ratio on the shear strength and the deformation capacity are demonstrated.

1. Introduction

The shear strength of reinforced concrete slabs with transverse reinforcement can be subdivided into a concrete and a steel contribution, whereas in particular for low transverse reinforcement ratios the concrete contribution becomes more important. Current design provisions are based on models which only consider the steel contribution [1,2] as well as such taking both contributions into account [3–5]; the semi-empirical concrete contribution refers to the modified compression field theory [6,7] as well as to the simplified modified compression field theory [8].

Neglecting the concrete contribution especially leads to an underestimation of the shear strength of slabs with low transverse reinforcement ratios. Based on experimental evidence, the question about the minimum transverse reinforcement that is required to enforce a ductile flexural failure has recently become of interest [9–13]. Next to the elimination of the brittle shear failure, tests on slabs with varying thickness [10,14] highlighted no size effect in shear in case that transverse reinforcement is provided.

On the basis of a sandwich model [1] for slab elements subjected to transverse shear forces as well as flexural and twisting moments, a new mechanical model for cracked, orthogonally reinforced concrete slab elements with and without transverse reinforcement was developed, the extended sandwich model ESM of Jaeger [11]. While the sandwich model [1] provides a limit analysis approach and the majority of the commonly used nonlinear FEM models take into account the influence of transverse shear forces insufficiently, the extended sandwich model enables a proper analysis of bending actions together with transverse shear forces. Bottom and top cover as well as the sandwich core are coupled by using a compatibility condition and the assumption of stressed crack faces in the core leads to a subdivision of the shear strength into a concrete and a steel contribution. A similar shear design model for beams and slabs with and without transverse reinforcement has been introduced in the latest Model Code for Concrete Structures [4,5]. In particular, the shear design provisions according Level III are directly comparable to the extended sandwich model. Local effects like punching due to concentrated shear forces in the vicinity of columns and wall-like columns are not treated and the beneficial effect of membrane forces is prudently neglected because of their sensitivity to unpredictable changes of the boundary conditions. With regard to additional theoretical investigations,

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http://dx.doi.org/10.1016/j.engstruct.2014.05.025
0141-0296/

the extended sandwich model in the present form can be completed for such cases. Compared to the latest Model Code for Concrete Structures [4], the shear design provisions for beams and slabs [5] and the design provisions for punching shear [15] are based on two different models. After a review of the basics of the sandwich model, the present paper describes the shear strength and the deformation behavior of reinforced concrete slabs with transverse reinforcement according to the new model. A comprehensive description of the flexural behavior as well as the treatment of shear forces in slabs without transverse reinforcement is presented in two companion papers [16,17].

2. Sandwich model

Slab elements are generally subjected to five stress resultants, namely the two flexural moments m_x and m_y, the twisting moments $m_{xy} = m_{yx}$ and the two transverse shear force components v_x and v_y as shown in Fig. 1(a). Introducing a sandwich model for orthogonally reinforced concrete slab elements [1], the covers have to resist to equivalent forces due to flexural and twisting moments, while the transverse shear forces are assigned to the core, see Fig. 1(b). The core thickness, d_v, is given by the distance between the median planes of the bottom and top cover, where z_B and z_T denote the effective thickness of the sandwich covers. Note that the core thickness correlates with the effective shear depth. The transverse shear forces v_x and v_y correspond to a principal shear force $v_0 = (v_x^2 + v_y^2)^{1/2}$ being transferred at an angle $\varphi_0 = \tan^{-1}(v_y/v_x)$ to the x-direction as shown in Fig. 1(c), while there is no shear transfer perpendicular to the direction of v_0.

Provided that the nominal shear stress $\tau_{z0} = v_0/d_v$ relative to the principal shear direction does not exceed a certain limit of about $f_{ct}/3$, the core is assumed to be uncracked, where f_{ct} = tensile strength of concrete. Thus, no transverse reinforcement has to be provided. The state of pure shear in the core leads to equal and opposite principal stresses with a magnitude of τ_{z0} and an inclination of $\psi_C = \pi/4$ to the in-plane of the slab, see Fig. 1(c), whereas the cover forces are not affected by the transverse shear force as shown in Fig. 1(b).

If the nominal shear stress relative to the principal shear direction exceeds a value of $\tau_{z0} > f_{ct}/3$, the core is considered to be cracked, and diagonal cracking occurs as depicted in Fig. 1(e). Thus, a transverse reinforcement corresponding to a reinforcement ratio of

$$\rho_z = \frac{v_0 \tan \Psi_{r0}}{d_v \sigma_{sz}} \tag{1}$$

has to be provided, where ψ_{r0} and σ_{sz} denote the inclination of the diagonal compressive stress field in the cracked core and the chosen

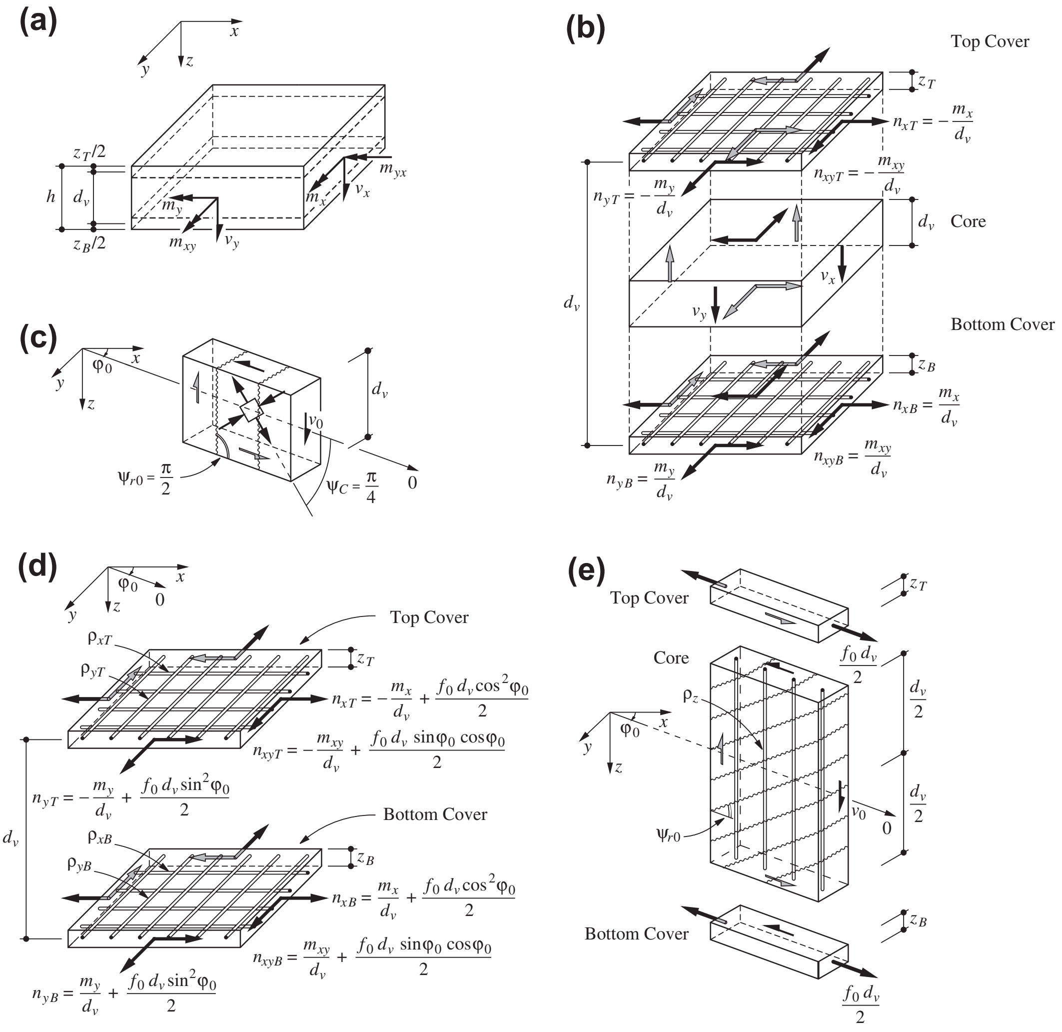

Fig. 1. Sandwich model: (a) stress resultants; (b) equivalent forces; (c) core element with disabled transverse reinforcement; (d) forces acting on cover elements in case of a core with enabled transverse reinforcement; (e) core element with enabled transverse reinforcement.