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Limit analysis of plane frames with piecewise linear hardening/softening behavior and axial–shear force–bending moment interaction

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ABSTRACT

Limit elastoplastic structural analysis is addressed herein in the context of mathematical programming aiming at determining the ultimate load capacity of frame structures at incipient collapse. Multi-segmental hardening/softening behavior of critical sections is incorporated in a direct and efficient manner in the yield condition. Axial–shear force–bending moment interaction is addressed referring to a nonlinear 3D yield surface, appropriately linearized to form a convex polyhedron. Equilibrium and compatibility requirements together with strength and complementarity constraints are used to formulate an optimization problem aiming at maximizing the loading factor. For every stress point and optimization iteration, a cone identification approach is proposed enabling the formulation of yield and complementarity conditions only for the specific targeted or activated yield hyperplane. The entire formulation is not affected by the linearization of either the yield surface or the constitutive relations and succeeds in reducing the size of yield and complementarity conditions to a minimum. Numerical results are presented that verify the validity and efficiency of the proposed method underlining the role of shear force interaction in specific cases.

1. Introduction

The direct determination of the ultimate limit state of a structure at incipient collapse subject to a permanent and a monotonically varying loading is a problem of primal interest in elastoplastic analysis of structures. In the past decades, this has been formulated within the context of mathematical programming offering a unifying approach for the limit analysis problems of structural plasticity.

This formulation considering piecewise linearization (PWL) of the yield criteria and constitutive relations has been introduced by Maier et al. [1–5], addressing both perfectly plastic and hardening structural behavior on the basis of holonomic or nonholonomic considerations. Piecewise linearization of yield criteria and constitutive laws allow for treating rigid-perfectly plastic limit analysis with unbounded ductility as a linear programming (LP) problem using the upper and lower bound theorems [6]. Adding in deformation constraints under holonomic assumption has generated a variety of alternative mathematical programming procedures, such as iterative Linear Programming, Quadratic Programming, Restricted Basis Linear Programming, Parametric Linear Complementarity and Parametric Quadratic Programming procedures for elastoplastic analysis of structures [6–9]. More recently, second-order cone programming (SOCP) problems have been proposed aiming at minimizing a linear function subjected to linear equality conditions and quadratic/conic inequality constraints. These SOCP problems were further generalized in the framework of semidefinite programming (SDP) [10,11].

Incorporation of deformation constraints and/or softening behavior under holonomic assumption for PWL yield and constitutive relations relies on complementarity conditions that change drastically the mathematical inner structure of the optimization problem converting it into a nonconvex one. This is known as Mathematical Programming with Equilibrium Constraints (MPEC) problem and various methods have been developed for its appropriate treatment [12–15]. Limit analysis of strain softening frames has been also thoroughly examined for holonomic [16–18] and non-holonomic behavior [19–21] under the effect of combined stresses (axial force–bending moment).

According to the standard formulation that addresses the holonomic elastoplastic problem, strength reserves are calculated for every cross section for all possible planes of the PWL yield surface. This generates for every cross section as many yield constraints as the number of yield planes. Moreover, under this consideration for all alternative planes of the PWL yield surface, incorporation of multi-linear hardening/softening in the yield condition engages all different segments of the constitutive relation. This increases

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the computational cost significantly, rendering real scale problems prohibitive. In addition, complementarity condition is perplexed considerably since the number of generated plastic multipliers for every cross section meets the number of all yield planes.

The aim of this work is to examine the shear force effect on the ultimate load carrying capacity, addressing a more general interaction within the stress resultants for holonomic behavior in frame structures. This is accomplished by reducing the complexity of the standard formulation and uncoupling the size of the problem from the multiplicity of linearization in 3D interaction surfaces. Elastoplastic analysis is dealt herein as an optimization problem with equilibrium, compatibility, yield and complementarity constraints on the basis of holonomic assumption. The adopted yield criterion accounts for the axial–shear force–bending moment interaction and is applied for isotropic piecewise linear hardening/softening behavior. The proposed formulation extends the concept of the identification of the critical yield line introduced for 2D interaction [22] into the yield hyperplane (cone identification) that corresponds to each cross section at every loading instance. Thus, the yield condition is formed only for one yield plane for each cross section and not for all existing ones. Moreover, piecewise linear hardening/softening behavior is incorporated with the number of linear segments not affecting the size of the problem.

The organization of the paper is as follows. First, the governing relations of holonomic elastoplastic problem are presented. Equilibrium, yield, kinematical and constitutive relations are stated and the concepts of cone and hardening/softening branch identification are elaborated. Then, the formulation of the elastoplastic analysis as a MPEC problem and its conversion into a nonlinear programming (NLP) problem is presented, incorporating the axial–shear force–bending moment interaction. Ultimately, numerical examples for steel frames are presented that illustrate the applicability of the proposed method and the role of shear force on the load carrying capacity of the structure.

Matrix notation is adopted throughout. Matrices are represented by capital bold-face letters, while vectors by lowercase bold characters.

2. Problem formulation

The entire formulation is based on the following assumptions. Plane frames consist of n straight prismatic elements, with n_f nodal degrees of freedom subjected only to nodal loading for reasons of simplicity. Frame displacements are assumed small enough so that the equilibrium equations refer to the initial undeformed configuration and plastic hinges are considered formed only at critical sections. Euler–Bernoulli or Timoshenko beam theory accounting for shear deformation effects is considered offering accurate stresses for regular and deep sections respectively. In both cases comparatively large shear forces may be induced that should be taken into account in the strength interaction. Apparent softening behavior (caused by local buckling, lateral–torsional buckling or by the semi-rigid nature of some steel connection types) is incorporated [17]. Yield functions and constitutive relations describing the cross-sectional behavior are beforehand appropriately linearized. Furthermore, a holonomic, i.e. path-independent, structural behavior is adopted. Although this is a simplified assumption, especially for the case of softening behavior, it can be considered reasonable for monotonically increasing loading [4,17,18]. For non-holonomic behavior a stepwise holonomic approach can be applied [21]. Moreover, isotropic hardening is adopted, which under monotonic loading and holonomic assumption yields satisfactory results [17].

For steel structures a 2D frame analysis is dominant. Ductile 3D moment resisting frames (MRFs) are rather unusual for steel buildings. In practice, ductile 2D MRFs in one direction are combined with either CBFs (concentrically braced frames) or EBFs (eccentrically braced frames) in the other direction [23]. However, especially for concrete structures a more realistic influence of shear interaction can be considered in the context of 3D frames under the effect of multi-stress component interaction (axial–shear force-biaxial moment interaction).

2.1. Equilibrium

From the six stress resultants developed at the ends of each beam element in a plane frame structure three are considered as independent, namely the axial force (s_1^i), bending moment at the start node j (s_2^i) and bending moment at the end node k (s_3^i), as shown in Fig. 1 and the remaining three are determined from equilibrium relations. The structural equilibrium relationship for the whole structure is then established as:

$$\boldsymbol{B} \cdot \boldsymbol{s} = a \cdot \boldsymbol{f} + \boldsymbol{f_d} \tag{1}$$

where $\boldsymbol{B}$ is the $(n_f \times 3n)$ structural equilibrium matrix, formed by assembling the corresponding element equilibrium matrices, $\boldsymbol{s}$ is a $(3n \times 1)$ vector for all primary stress resultants, a is a scalar load factor, $\boldsymbol{f}$ is the $(n_f \times 1)$ basic monotonically varying nodal forces and $\boldsymbol{f_d}$ is the $(n_f \times 1)$ fixed nodal load vector.

2.2. Piecewise linear yield condition

A yield criterion expressed in normalized stress space $n - v - m$ (i.e. normalized axial force with respect to axial yield limit, normalized shear force with respect to shear yield limit and normalized bending moment with respect to bending moment yield limit) delimits the elastic region and designates the stress evolution in the plastic region for perfectly plastic or hardening/softening behavior. The nonlinear yield surface is herein "a priori" linearized to express the yield conditions in the form of linear constraints. The standard formulation involves all the hyperplanes that discretize the yield surface. This increases considerably the number of yield constraints per critical section, complicating also incorporation of multi-linear hardening/softening behavior. However, the only information needed is the targeted or activated hyperplane for every stress point per critical section, generated at every optimization step. This is accomplished through the proposed identification process (Section 2.2.1) that detects the specific cone in which every stress point resides. Each identified cone is associated with only one yield hyperplane and thus a single yield constraint for each critical section is implemented. This consideration reduces the number of yield constraints retaining only those

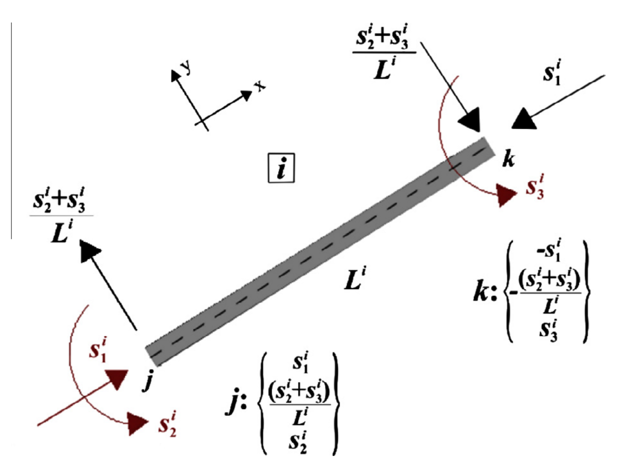

Fig. 1. Frame element i with equilibrated stress resultants-end actions.