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Distortional theory for the analysis of wide flange steel beams

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ABSTRACT

A distortional theory is developed for the analysis of doubly symmetric and mono-symmetric wide flange beams under general loading. The governing differential equations of equilibrium and associated boundary conditions are derived based on the principle of potential energy. The theory captures shear deformation effects in the web and local and global warping effects. In contrast to classical beam theories, the present study captures web distortion by accounting for its flexibility within the plane of the cross-section while considering the flanges as Euler–Bernoulli beams. The formulation yields two systems of coupled differential equations of equilibrium in seven displacements fields. The first system governs the longitudinal transverse response and involves three displacement fields, and the second system governs the lateral torsional response and involves four displacement fields. Closed form solutions are then developed for both coupled systems under general loading. Numerical solutions for practical problems are then provided to illustrate the applicability of the formulation. Comparisons to results based on 3D shell finite element solutions show the validity of the results. The theory preserves the relative simplicity of one dimensional beam theories while effectively capturing the three-dimensional distortional phenomena normally captured within computationally expensive 3D FEA.

1. Introduction and literature review

Conventionally, hot rolled and welded wide flange steel beams are analyzed using the Vlasov theory (Vlasov [1]) which neglects distortional effects within the cross-section. Given that such cross-sections are relatively thick compared to cold-formed steel sections, it is often accepted that the Vlasov theory should reasonably predict their response. This is generally the case for a large number of applications. Nevertheless, some exceptions to this general rule are known to exist. This includes cross-sections with slender webs and thick flanges, short-span members, and/or when the applied loads are localized such that they deform a localized portion of the cross-section. In such cases, the web can undergo significant distortion and the application of the Vlasov theory or a variation thereof would lead to unreliable response predictions. The designer would typically need to resort to shell finite element analysis for an accurate response prediction. Another alternative is also available through various distortional beam theories which have developed. A common theme among these theories has been to attempt to uncouple the distortion deformation from other conventional deformation modes, resulting in simple uncoupled equilibrium equations. The associated orthogonalization process is typically rather involved. Within this context, the present paper aims at developing a new distortional theory for wide flange sections. Rather than attempting to uncouple the resulting differential equations of equilibrium, the study develops a fully coupled system of governing equations and then provides a general closed form solution for the resulting system.

Classical theories such as the Euler–Bernoulli beam, the Timoshenko Beam (Timoshenko [2], the thin walled beams Vlasov [1] and Gjelsvik [3] are based on the common assumption of neglecting cross-sectional distortional effects. More modern beam theories for the static analysis of beams which accounting for distortion include the work of Schardt [4] who developed a Generalized Beam Theory (GBT), mostly for very thin walled members (e.g., cold form cross sections), in which he categorized the behavior of the beam into: (a) the four classical modes of deformation analogous of the Vlasov beam (Vlasov [1]), i.e., axial, biaxial bending and twist), and (b) distortion of the cross section in its own plane. His theory introduces distortional modes and associated warping functions that are orthogonal to the other four classical kinematic modes of deformation and thus consists of five uncoupled differential equations in four classical displacement fields in addition to a distortional field. Davies and Leach [5] applied the GBT theory to cold formed sections. Using closed form solutions and the finite difference method, they solved the fourth order differential equation of distortion. Jönsson [6] postulated that

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distortion has commonalities with torsion and thus leads to analogous internal forces. He extended the concept of sectorial coordinates, previously introduced by Vlasov, to characterize distortional deformation. He developed a one-dimensional finite element to determine the torsional and distortional warping functions and corresponding shear distribution within the cross-section. In two subsequent papers, Jönsson [7,8], combined the outcome of his previous study with the principle of stationary potential energy to formulate the distortional warping functions and corresponding shear stresses for open and closed sections. The resulting torsional and distortional equilibrium equations are observed to be generally remain coupled. In another treatment, Rendek and Baláž [9] adopted vector analysis to simplify the orthogonalization process between displacement modes normal to the cross-section and determine the associated modal loads and sectional properties. They approximated the transverse displacements by Hermitian cubic functions of the longitudinal coordinate and quantified the transverse bending moments and distortional sectional properties. Based on a modified version of Prokic theory, Saadé and Warzée [10] developed a finite element which accounts for cross-sectional distortion. Later on, Gonçalves and Ritto-Corrêa [11] devised a new approach to determine the deformation modes in which they subdivided the space of possible solutions into four sub-spaces based on four distinct kinematic hypotheses. Under each hypothesis, they identified the warping and in-plane modes. An orthogonalization process is then performed on the associated modes to determine the relevant cross-sectional constants. In their study, local deformations and shear deformations are also captured by introducing intermediate nodes. Jönsson and Andreassen [12] developed a GBT solution in which they related the displacements within the plane of the cross section to the nodal displacements through Hermitian polynomials, thus capturing distortional effects. They orthogonalized the various modes by enforcing kinematic constraints, ultimately leading to two differential equations per segment which characterize distortion within the segment. Using polynomial interpolations of the longitudinal coordinate for conventional modes and exponential interpolations for distortional modes, they developed a finite element that captures distortion. In a more recent study, Andreassen and Jönsson [13] generalized their previous work by incorporating the effect of distributed loads.

A common theme among the above studies is the various elaborate techniques devised by authors to uncouple the resulting field equations governing distortion from the remaining classical modes of deformation, culminating in simpler field equations. The present theory deviates from the previous convention by avoiding the complications involved in the uncoupling process of the governing field equations, thus yielding a highly coupled system of equations, but then providing closed form solutions for the resulting systems of equations under general loads.

Distortional effects also have received considerable attention in buckling investigations of thin-walled beams. This includes the work of Rajasekaran and Murray [14] who developed a buckling finite element for wide flange beams based on overlaying the kinematics of the Vlasov thin walled beam theory on those of the Kirchhoff plate bending theory to capture the distortional behavior of web and flanges. Johnson and Will [15] developed a shell buckling finite element with 22 DOFs and applied it to capture the distortional buckling behavior and predicting the buckling capacity of wide flange beams and columns. In a subsequent study, using the finite strip method, Hancock [16] has shown that, in beams with slender webs and stocky flanges, the web is susceptible to distortion while the flanges remain undistorted. Hancock and Trahair [17] developed an approximate distortional buckling solution for simply supported wide flange beams under uniform moments and axial forces. In their solution, they assumed that the web deforms as a cubic function along the height while both flanges undergo distinct angles of twist. Bradford and Trahair [18] developed a finite element for I-shaped members under unequal end moments and compressive axial forces. Their element is based on Hermitian shape functions to interpolate the displacements along the span. Bradford and Trahair [19] extended their work to other thin-walled sections such as lipped channels by retaining the flexible web assumption. Using a plate bending formulation, Roberts and Jhita [20] formulated the buckling characteristic equation of wide flange sections. In a series of papers, Bradford extended his previous work to develop distortional lateral buckling solutions for mono-symmetric cross-sections [21], inelastic buckling of hot-rolled I-beams [22], mono-symmetric I-beams with flanges restrained by continuous lateral elastic restrains [23], beam columns [24], cantilevers [25], investigating the effects of end conditions, rotational and translational restraints for I-beams [26], and beams laterally restrained at one flange [27]. A common feature in all Bradford's work is the approximation of web lateral displacements through a cubic function. Based on the Rayleigh Ritz energy method, Wang and Chin [28] provided a comparison between distortional and non-distortional buckling for mono-symmetric simply supported beam-columns. Hughes and Ma [29] developed a distortional buckling solution for simply supported mono-symmetric I-beams under point loads and extended their work [30] to beams under distributed transverse loads and unequal end moments. In their study, they assumed that the web distorts as a fifth order polynomial function. Dekker and Kemp [31] developed a distortional buckling solution in which they idealized the flanges of an I-Beam as translational and rotational springs, while the flexible web remains as an elastic plate. In a study on simply supported I-beams, Pi and Trahair [32] developed a technique to quantify the torsional and warping rigidities which incorporates distortional effects. Their solution has incorporated pre-buckling and end-warping restrains effects. Ng and Ronagh [33] developed a Fourier series solution to obtain the distortional buckling capacity of doubly-symmetric I-Beams, which models the effect of elastic restraints and loading offset from the shear center. Based on shell FEA, Samanta and Kumar [34] investigated the distortional buckling of simply supported mono-symmetric I beams. Vrcelj and Bradford [35] investigated the effect of lateral and rotational restraints on the distortional buckling of I-beams with a tension flange seated on a support. Ádány and Schafer [36] devised a mode decomposition technique within the constrained finite strip method to extract distortional buckling moments. Ádány and Schafer [36], Samanta and Kumar [37] extended their previous work to cantilevers of mono-symmetric I cross-sections and investigated the effect of load position and bracing height on the distortional buckling capacity. Using the finite strip method, Zirakian [38] studied the distortional buckling of doubly symmetric I-beams. He showed that AISC [39] recommendations for lateral torsional buckling provisions of beams with slender webs, which neglects the Saint-Venant torsional stiffness, provide overly conservative means of incorporating the effect of distortion. By assuming a quadratic distribution of the web lateral displacement, Chen and Ye [40] developed the potential energy expression for the distortional buckling of I-beams and applied the Ritz method to determine the buckling solution for simply supported beams with a single restrained flange. Using the effective section properties developed by Pi and Trahair [32], Kalkan and Buyukkaragoz [41] determined distortional critical moments and compared their results to those in AISC [39], EC3 [42], and AS 4100 [43]. Their study involved distortional buckling moments in the elastic and inelastic ranges.

A comparative summary for the above studies is provided in Table 1.

The majority of the above buckling studies assume that the lateral displacement of the web to have a cubic distribution along the height, while the flanges remain undistorted and having distinct