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A numerical investigation into the plastic buckling paradox for circular cylindrical shells under axial compression

Rabee Shamass [a,*], Giulio Alfano [a], Federico Guarracino [b]

[a] School of Engineering and Design, Brunel University, UB8 3PH Uxbridge, UK
[b] University of Naples 'Federico II', Via Claudio 21, 80125 Napoli, Italy

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ABSTRACT

It is widely accepted that for many buckling problems of plates and shells in the plastic range the flow theory of plasticity leads to a significant overestimation of the buckling stress while the deformation theory provides much more accurate predictions and is therefore generally recommended for use in practical applications. The present work aims to contribute to further understanding of the seeming differences between these two theories with particular regards to circular cylindrical shells subjected to axial compression. A clearer understanding of the two theories is established using accurate numerical examples and comparisons with some widely cited accurate physical test results. It is found that, contrary to common perception, by using a geometrically nonlinear finite element formulation with carefully determined and validated constitutive laws very good agreement between numerical and test results can be obtained in the case of the physically more sound flow theory of plasticity. The reasons underlying the apparent buckling paradox found in the literature regarding the application of deformation and flow theories and the different conclusions reached in this work are investigated and discussed in detail.

1. Introduction

Plastic buckling of circular cylindrical shells has been the subject of active research for many decades due to its importance to the design of aerospace, submarine, offshore and civil engineering structures. It typically occurs in the case of moderately thick cylinders subjected to axial compression, external pressure, torsion or combinations of such loads. For example, buried pipelines used to transport fluids or pipelines resting on a deformed foundation can undergo high compressive axial loads, which can lead to axial buckling, or experience high external pressure leading to ovalisation buckling.

In general, the numerical analysis of plastic buckling of practical cylinders requires the determination of the nonlinear load–deflection path and must also consider bifurcation and mode changes. Therefore, an accurate prediction of the critical loads in the plastic range requires accounting for moderate large deflection and, one would expect, nonlinear, irreversible, path-dependent material behaviour [5].

On the other hand, path-dependence is not always invoked as a necessary hypothesis for modelling purposes. In fact, based on whether path-dependence is accounted for or not, the plasticity models that have been proposed for metals in the strain hardening range can be divided into two groups: the 'deformation theory' of plasticity and the 'flow theory' of plasticity. In both of these theories the plastic deformations do not allow volume changes as plastic yielding is governed by the second invariant J_2 of the deviatoric part of the stress tensor, whereby in this respect they are both so-called J_2 theories. However, the deformation theory of plasticity is based on the assumption that for continued loading the state of stress is uniquely determined by the state of strain and, therefore, it is a special class of path-independent non-linear elasticity constitutive laws. According to this assumption, after a strain reversal, rather than recovering the initial elastic stiffness, as is found in physical tests, the initial loading curve is followed. On the other hand, the flow theory of plasticity assumes that an (infinitesimal) increment of stress is uniquely determined by the existing strain and its increment. This leads to a path-dependent relationship in which the current stress depends not only on the value of the current total strain but also on how the actual strain value has been reached, thus making the constitutive relationship path dependent.

There is a general agreement among engineers and researchers that the deformation theory of plasticity lacks physical rigour in comparison to the flow theory. Use of the deformation theory predicts buckling loads that are less than corresponding loads obtained with the incremental theory, and evidence of comparison between measured and calculated buckling loads points in favour

* Corresponding author. Tel.: +44 7411979619.
E-mail addresses: Rabee.Shamass@brunel.ac.uk, rabeeshammas@yahoo.com (R. Shamass).

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of deformation theory results. This fact may be related to several factors and Onat and Drucker [15] first pointed out through an approximate analysis that buckling predictions based on the flow theory for long plates supported on three sides tend to those predicted by the deformation theory if small but unavoidable imperfections are taken into account. Later, this so called "plate buckling paradox" was theoretically examined by Sewell [18] who obtained lower flow theory buckling loads by allowing a variation in the direction of the unit normal. In a subsequent study Sewell [19] also illustrated that the use of Tresca yield surface in the flow theory of plasticity leads to significant reductions in the buckling loads. An extensive discussion of the buckling paradox as understood at that time was given by Hutchinson [10].

More recently, Wang and Huang [22] examined the elastoplastic buckling of a rectangular plate made of alloy Al 7075 T6, typically used in the aerospace industry, subjected to biaxial loading (uniform compressive load $\sigma_2 = -\sigma$ in one direction and tension or compression load $\sigma_1 = \xi\sigma$ in the perpendicular direction, where ξ is a constant). A detailed parametric study was made using the differential quadrature method (DQ) and the authors concluded that the small deformation assumption used to establish the governing differential equation could possibly be the reason for the large discrepancy between the results obtained using either deformation or flow theory. In a later paper, Wang and Zhang [23] used the DQ method to obtain the elastoplastic buckling stresses for thick rectangular plates with various values of the thickness-to-side-length ratio, and for various material properties and boundary conditions. They found that the discrepancy in the calculated buckling stresses between the two theories of plasticity gets larger with increasing plate thickness, the ratio E/σ_y and exponent n in the Ramberg–Osgood expression, where E and σ_y are the Young's modulus and yield strength. They suggested that another explanation of the discrepancies in the results using the two theories for thick plates could be that the deformation theory predicts an increasingly lower in-plane shear modulus as the level of plasticity increases, which results in lower calculated buckling-stress values.

Restricting attention to the plastic buckling of circular cylindrical shells, Mao and Lu [12] analytically examined simply supported cylinders made of aluminium alloy subjected to axial compression load. They compared the buckling stresses predicted by their analytical formula with the experimental results conducted by Lee [11] and found that the deformation theory provides closer results with the tests while the flow theory significantly over-predicts the critical loads.

Blachut et al. [3] conducted experimental and numerical analyses of 30 mild-steel machined cylinders, of different dimensions, subject to axial tension and increasing external pressure. They showed that agreement between the buckling stresses calculated using the two theories was strongly dependent on the ratio of the length L of the cylindrical shell to its outer diameter D. For short cylinders ($L/D \leqslant 1$) the plastic buckling pressure predicted by flow or deformation theory coincided only when the tensile axial load vanished. By increasing the axial tensile load, the plastic buckling pressure calculated using by the flow theory of plasticity quickly diverged from corresponding values calculated using the deformation theory, which were closer to the experimental values. For specimens with L/D ranging from 1.5 to 2 the results predicted by both theories were very similar for a certain range of combined loading, beyond which the values calculated using the flow theory began to deviate from the corresponding results using the deformation theory and became unrealistic in correspondence of large plastic strains.

Ore and Durban [16] analytically investigated the buckling of axially compressed circular cylindrical shells in the plastic range for various boundary conditions. Similar to Mao and Lu [12], they concluded that the buckling compression stresses predicted by the deformation theory appeared to be in good agreement with measured test results, while those provided by the flow theory overestimated the measured test values. Moreover, the authors observed that the differences between the theoretical results predicted by the flow and deformation theory reduced with increasing value of the strain hardening parameter.

Bardi and Kyriakides [2] tested fifteen cylindrical stainless steel tubes, with D/t ranging between 23 and 52, under axial compression and determined the critical stresses and strains at the onset of wrinkling. They reported the buckling modes, including the number and the size of waves. They also calculated the same quantities analytically using the deformation or the flow plasticity theory. The calculations included the effects of assuming both isotropic and anisotropic material behaviour. Bardi and Kyriakides concluded that the flow theory significantly over-predicts the critical stresses and strains while the deformation theory leads to critical stress and strain in better agreement with the experimental results. Moreover, the flow theory grossly over-predicted the wavelength of wrinkles while the deformation theory was in better agreement with the wavelengths measured in the tests.

The plastic paradox does not seem to be limited to the buckling of plates and cylinders. For example, Galletly et al. [4] investigated the plastic buckling of six machined steel torispherical domes of different geometries and subjected to internal pressure. The tests were carried out to highlight the differences in buckling stresses calculated, using the code BOSOR 5 [6], with either the flow or the deformation theory. They measured low-amplitude waves in the knuckle of the torispherical domes by probes allocated at the knuckle region for all six specimens. These waves grew with the increasing internal pressure in four test specimens and became visible to the naked eye while in other two specimens the waves could not be visually detected but could be felt by finger-tip contact. In their analysis they found that, for all the tests, the buckling mode failure and the internal pressure predicted by the deformation theory was in good agreement with the experimental results, the difference varying between 6% and 29%. On the other hand, the flow theory did not predict a buckling failure mode for any of the four test specimens.

In the framework provided by the above cited publications, the present work aims to shed light on the plastic buckling paradox by conducting accurate linear and nonlinear finite-element modelling of buckling of cylindrical shells using the flow theory and the deformation theory of plasticity.

Attention is focused on cylindrical shells subject to axial compression with outer-radius-to-thickness ratio R/t ranging between 9 and 120, because of the great significance of this geometry and loading conditions for engineering application. The predictions have been compared with widely recognised experimental results reported in the literature by Lee [11] and Batterman [1] and with the analytical results reported by Mao and Lu [12] and Ore and Durban [16].

It is found that, in contrast to common understanding, by using carefully validated geometrically nonlinear finite element (FE) modelling a very good agreement between numerical and experimental results can be obtained in the case of the physically sound flow theory of plasticity. The reasons underlying the apparent buckling paradox are then investigated and discussed in detail.

2. Test samples and finite-element modelling

2.1. Geometry and elements

The plastic buckling of perfect and imperfect cylinders subjected to axial compression has been numerically simulated