Engineering Structures 75 (2014) 577–589

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Analytical uncertainty quantification for modal frequencies with structural parameter uncertainty using a Gaussian process metamodel

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ARTICLE INFO

Article history:
Received 14 July 2013
Revised 13 June 2014
Accepted 18 June 2014
Available online 11 July 2014

Keywords:
Uncertainty quantification
Structural dynamics
Modal frequency
Monte Carlo simulation
Gaussian process Metamodel
Bridge structure

ABSTRACT

Quantifying the uncertainty in the dynamic properties of large-scale complex engineering structures presents significant computational challenges. Monte Carlo simulation (MCS) method is extensively employed to perform uncertainty quantification (UQ) because of its generality, stability, and easy implementation. However, a brute-force MCS approach may be unaffordable and impractical when the target model contains a large number of uncertain parameters. In this circumstance, MCS requires a potentially burdensome (if not computationally intractable) number of model evaluations to obtain a credible estimate of the global statistics. In this study, a general framework for analytical UQ of model outputs using a Gaussian process (GP) metamodel is presented, where case inputs are characterized as normal and/or uniform random variables. A detailed derivation of important low-order statistical moments (mean and variance) is given analytically. This analytical method is adopted to characterize the uncertainty of modal frequencies of two bridges with assumed normally- and uniformly-distributed parameters. Meanwhile, the brute-force MCS approach is used for comparison of GP metamodel-derived statistics. Results show that the GP method outperforms the MCS methodology in terms of computational cost, with consistency in the "true" values obtained by MCS. It demonstrates that this GP method is feasible and reliable for modal frequency UQ of complex structures.

1. Introduction

Uncertainty is ubiquitous in all sorts of structural analysis and system identification applications. Specifically, as-built engineering structures are inevitably subjected to many sources of variability, including but not limited to manufacturing-induced geometric tolerances, inherent random variation of materials, imprecisely controlled boundary conditions, load variation, and ambient temperature fluctuation. In the risk assessment community, uncertainties are classified into two categories: *aleatory* and *epistemic*, according to their fundamental essence [1]. The former (also termed stochastic, irreducible, or type A uncertainty) is the uncertainty stemming from inherent variation or randomness, whereas the latter (also termed subjective, reducible, or type B uncertainty) is the uncertainty due to incomplete information. Understanding these sources of uncertainty plays an important role in dealing with it, because different types of uncertainty call for different methods of treatment. For a comprehensive understanding of uncertainty sources and the associated management methods, see O'Hagan [2], Roy and Oberkampf [3], and Liang and Mahadevan [4].

The quantification of uncertainty in structural dynamic properties remains an important topic, as such properties are used in a variety of important decision-making applications, such as structural health monitoring, model updating, or test validation/verification. Such dynamic properties include (but are not necessarily limited to) natural frequencies, mode shapes, the frequency response function (FRF). To obtain more credible predictions for dynamic properties or for proper use in hypothesis testing, the uncertainty present in them should be taken into account. Among these various properties, there is a large body of research work regarding the uncertainty quantification of the FRF. This is due to the fact that FRF has clear physical interpretation and does not require a modal analysis; thus modal parameter identification errors are eliminated. Moreover, FRF data are much more easily accessible than other dynamic properties. The approaches exploited by researchers to characterize uncertainty of the FRF consist of MCS [5], random matrix theory Soize [6], fuzzy set theory [7], interval analysis [8], MCS in conjunction with

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http://dx.doi.org/10.1016/j.engstruct.2014.06.028
0141-0296/

metamodeling technique [9,10], statistical modeling approach [11]. Modal frequencies are an indispensable structural dynamic quantity that provides global resonant information about the structure, and because they are relatively easily and robustly measured and relatively low-dimensional, they are often used in the areas of FEM updating [12–14] and damage identification [15]. Modal frequencies also play a critical role in many structural design processes. For example, if the modal frequencies of a pedestrian bridge fall in the range of human movement frequencies, the human-induced vibration will make pedestrians uncomfortable and even cause safety problems. This paper will thus focus on the quantification of modal frequencies where uncertainty is present in the structure. The uncertainties that have a considerable impact on the modal frequencies include parameter uncertainty, boundary condition variability, and temperature fluctuation. In this work, we restrict our scope on parameter uncertainty, which is the most studied type of uncertainty.

The aforementioned random matrix theory, fuzzy set theory, and interval analysis require the structural stiffness and mass matrices for the subsequent UQ of structural dynamics. For large-scale complex civil structures, they are usually modeled by the high-resolution FEMs involving up to tens or hundreds of millions complex elements with the aid of commercial finite element analysis (FEA) packages, such as ANSYS, ABAQUS and SAP2000. Extracting stiffness and mass matrices from these FEA packages is not an easy task, and multiple iterations between numerical software (e.g., MATLAB) and FEA package will increase computational cost in some degree. The MCS is the most commonly-used and well-known method for uncertainty quantification and propagation. Its robustness depends on neither the type of problem nor resident dimensionality. Furthermore, MCS for UQ also have the advantage of easy implementation. The statistical properties of the model outputs can be obtained by just performing repeated model evaluations using random or pseudorandom numbers to sample from probability distributions of model inputs. However, MCS is extremely time-consuming, because it requires a large number of model evaluations to characterize statistical properties of the model outputs. The behavior of most real structures is simulated by a finite element model (FEM) with a large number of elements (up to tens or hundreds of millions) and complex elements (e.g., solid and shell elements). Thus, when applied to execute UQ tasks for these structures, the brute-force MCS method directly using computationally-expensive FEM will be limited in its applicability due to the high computational cost. For example, in the year 2000, Los Alamos National Laboratory (LANL) in the US quantified the propagation of uncertainties through a nonlinear FEM simulation of a blast-loaded structure on one of the worlds most powerful computers at that time, Blue Mountain. The analysis took over 72 h and would have required 17.8 years of equivalent single-processor time [16]. Accordingly, the brute-force MCS for UQ is unaffordable and thus unfeasible, especially for the complex physical system.

For the sake of reduction in computational burden, several researchers use the fast-running GP metamodel (also called *Kriging* process) as the surrogate model for the more computationally-expensive simulation of the complex physical system in order to facilitate the daunting task of UQ. DiazDelaO et al. [9], and Xia and Tang [10] explore the application of GP metamodel-based MCS for the UQ of FRFs propagated from uncertain parameters. Lockwood et al. [17] utilize gradient information-assisted GP model (usually known as Gradient-Enhanced Kriging, GEK) to reduce the computational cost associated with MCS for UQ in viscous hypersonic flows. Within GEK framework, Dwight and Han [18] employ sparse grid integration to perform UQ in Computational Fluid Dynamics (CFD). Using gradient information and prediction uncertainty of GP model, Shimoyama et al. [19] develop a dynamic adaptive sampling scheme for efficient UQ. Liu and Göörtz [20] combine GEK with Niederreiter sequence based quasi-Monte Carlo (QMC) quadrature to conduct the task of UQ in CFD model subjected to geometric uncertainty. To sum up, these methods all use GP metamodel-based MCS to carry out UQ with computational time reduced, and some of them introduce gradient information, sparse grid integration, dynamic adaptive sampling scheme, or QMC quadrature to further accelerate UQ. This paper proposes an analytical method for UQ of modal frequencies using a GP metamodel, with the specific formulation that the uncertain parameters are either uniformly- or normally-distributed. In the machine learning literature, Girard et al. [21] have used a GP metamodel with squared exponential covariance function to make analytical predictions several time steps ahead for normally-distributed uncertain inputs. We adopt and extend Girard et al.'s method, and use a GP metamodel to quantify the uncertainty of modal frequencies from uniformly- and/or normally-distributed parameters. This present analytical GP metamodel approach is more efficient and accurate than GP metamodel-based MCS because it conducts the task of UQ in an analytically integrated manner. It should be pointed out that if the uncertain parameters do not follow (or cannot be modeled with) normal or uniform distributions, using efficient sampling-based MCS method with a GP metamodel is also a promising alternative to alleviate the computational burden associated with UQ drastically.

This paper makes the following main contributions: (1) We develop the analytical GP metamodel-based method for UQ in structural dynamics of cases whose parameters are specified by normal and/or uniform distributions, which are the two most useful (and commonly-used) distributions to characterize parameter uncertainty in the engineering community, absent very domain-specific knowledge. (2) We use the brute-force MCS as a benchmark to verify the feasibility of the proposed method in terms of computational accuracy and efficiency. (3) We explore the relationship between the uncertainty magnitude of individual parameter and the induced overall uncertainty in structural dynamics. Thereby, we can have a good understanding of relative importance of uncertainty magnitude of individual parameter to the uncertainty in structural dynamics.

The rest of this paper is organized as follows. Section 2 outlines the formulation of GP metamodel, and then Section 3 describes the proposed analytical uncertainty quantification using the GP metamodel. The application of the GP approach to UQ of modal frequencies of two real-world bridges is presented in Section 4. Finally, Section 5 concludes this work.

2. Gaussian process metamodel

Gaussian process (GP) metamodel is developed based on concepts of Bayesian statistics. The probabilistic, non-parametric GP model is favored here because of its flexibility in representation of a complex physical system and its ability to quantify the uncertainty associated with its prediction. The flexibility enables a GP metamodel not to be restricted to a certain functional form due to a wide range of covariance functions. The application of GP metamodel to deterministic computer code simulators dates back to the work of Sacks et al. [22]. Recently, there has been an increasing application of GP metamodel in engineering area (e.g., Jones et al. [23], Simpson et al. [24], Dwight and Han [18], Lockwood et al. [17], Khodaparast et al. [25], Sankararaman and Mahadevan [26], Becker et al. [27], DiazDelaO et al. [9], Xia and Tang [10], Shimoyama et al. [19], Liu and Göörtz [20], Wan and Ren [28], and the state of the art may be found in the following Refs.: Mackay [29], Santner et al. [30], and Rasmussen and Williams [31].