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Structural reliability analysis based on probabilistic response modelling using the Maximum Entropy Method

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ABSTRACT

This paper investigates the efficiency of different approaches for structural reliability analysis with implicit limit state functions. The focus is on techniques that provide the probabilistic description of response quantities obtained by non-linear Finite Element Analysis (FEA) that are then used in the reliability assessment. Using this approach, statistical moments of the response and probabilities associated with specific failure events can be calculated, both of which are of interest in most probabilistic mechanics applications. In this paper, an efficient method for response variability and reliability analysis based on FEA is proposed. The approach combines the Maximum Entropy Fitting Method (MEM) for probabilistic modelling of the response and the First Order Reliability Method (FORM) for reliability evaluation. The results obtained by the MEM are compared with the ones obtained by an approach also available for probabilistic characterization of the response that consists in fitting a 3-parameter lognormal distribution to three fractiles of the response calculated by inverse FORM. The reliability analysis is performed using FORM with explicit limit state functions derived from the different probabilistic models of the response. These results are also compared with the ones obtained using FORM directly linked to the FEA and the Response Surface Method (RSM). The proposed approach is first applied to a simple elastic problem of a portal frame and then applied to the analysis of a ship stiffened plate subjected to longitudinal compression and lateral pressure.

1. Introduction

In recent decades, the design of marine structures has become more focused on the probabilistic limit state design, which has been successfully adopted in civil engineering and other domains. The key problem in the modern structural design is to properly account for the uncertainties on the loads and on the material and geometrical properties when analysing and designing complex structural systems [1].

The structural analysis under uncertainty is the main task of the stochastic or probabilistic mechanics, which has developed fast in the last decades mainly focused on the use finite element approaches in connection with stochastic or probabilistic methods [2]. The existing methods for stochastic mechanics approaches can be classified with respect to the type of results they primarily yield. Two categories of methods can be distinguished: (i) those aimed at estimating the statistical description of response quantities (usually the mean and variance), which are normally called second-moment methods, and (ii) those aimed at quantifying the probability associated with a specified failure criteria denoted as reliability methods.

The structural reliability methods are concerned with the assessment of the probability of a limit state violation of a structural component or system at any stage in his life. With the limit state function expressed as $g(\mathbf{X})$, the failure probability (P_f) is defined as the probability of occurrence of the event $g(\mathbf{X}) \leqslant 0$,

$$p_f = P[g(\mathbf{X}) \leqslant 0] = \int_{\Omega_f} f_{\mathbf{X}}(\mathbf{x})dx \tag{1}$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function for the n-dimensional vector ($\mathbf{X}$) of random variables $\{X_1, X_2, \ldots, X_n\}$ that represent the uncertainties on the loads and on the material and geometrical properties of the structure. The region of integration, $\Omega_f = \{x : g(\mathbf{x}) \leqslant 0\}$, denotes the failure domain that corresponds to the space of limit state violation. Except for some special cases, the integration in Eq. (1) over the failure domain cannot be performed analytically. Typically, p_f is estimated by Monte Carlo (MC) simulation or using approximate methods such as the First (FORM) or Second (SORM) Order Reliability Methods. A detailed

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description of these methods as well as other possible methods used for reliability assessment can be found e.g. in [3,4].

In most of practical applications involving complex structures, $g(\mathbf{X})$, is defined implicitly, i.e., its evaluation requires numerical response calculations by means of Finite Element Analysis (FEA). There are several different possible methods for reliability analysis with implicit limit state functions (LSF). These methods may be classified into three main categories as follows [5]: (1) Direct Methods (DM); (2) Limit State Approximation that includes Response Surface Methods (RSM) and Artificial Neural Network Methods (ANNM), (3) Statistical Response Characterization Methods (SRCM) that include the use of the Spectral Stochastic Finite Element Method (SSFEM) proposed by Ghanem and Spanos [6] for probabilistic representation of response quantities and expanded to various applications as for example in [7].

The Direct Method is a relatively accurate way to compute the reliability of problems with implicit limit state functions. The approach uses FORM/SORM or MC simulation methods and the calls of limit state function are transferred to the FEA, and the results are passed back to the reliability processor, as done by Teixeira and Guedes Soares [8]. The reliability processor has a built-in FEA code or is linked to an external software (such as ANSYS) to invoke a FEA of a predefined model with new values of random variables. The computational effort depends on the complexity of FEA, type of analysis and on the number of random variables. When using FORM, the gradients required in the reliability analysis are often calculated by a finite difference method.

One way of reducing the computational cost in the structural reliability analysis based on FEA is to find an approximate response function that replaces the true implicit limit state function. This well-established approach is known as Response Surface Method (RSM). For the feasibility of Response Surface approaches, it is essential to reduce the number of variables to a tractable amount. The response surface method is thought to be the best way to solve the reliability problems that involve limit state functions that are not expressed explicitly. Several response surface models suitable for the application in structural reliability have been introduced and compared by Bucher and Most [9]. Typically, a general quadratic polynomial is used as approximation function, with first-order, second-order and cross terms of the n random variables, given by:

$$Y = c_0 + \sum_{i=1}^{n} c_i X_i + \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} X_i X_j \tag{2}$$

where c_i, $i = 0, 1, \cdots, n$ and c_{ij}, i, $j = 0, 1, \cdots, n$ are the coefficients which can be evaluated by regression analysis such as the least-squares method.

Kmiecik and Guedes Soares [10] have used the RSM for probabilistic modelling of the strength of compressed steel plates and, recently, Teixeira and Guedes Soares [11] extended the use of this technique to reliability problems involving random fields of corrosion. Gaspar et al. [12] have combined a response surface approach with a Monte Carlo based simulation method to efficiently solve structural system reliability problems that involve nonlinear finite element analysis, while in [13] Gaspar et al. opted for the use of a more simplified structural analysis method. As an alternative to the traditional RSM based on first- or second-order polynomial, Kriging models (e.g. [14]) and Artificial Neural Networks (ANN) algorithms (e.g. [15–18]) introduced for universal function approximations have also been used for structural reliability assessment by several researchers. Bucher and Most [9] have applied these approximation methods to several examples of nonlinear structural analysis concluding that the relative accuracy of the various approaches depends on the specific problem under consideration.

The Statistical Response Characterization Methods (SRCM) typically use explicit limit state functions in which the response is represented by a random variable with probability density function (PDF) previously derived. The reliability analysis is then performed using Monte Carlo simulation or FORM/SORM methods ([5]). Using this approach, attempts have been made to extend the Spectral Stochastic Finite Element Method (SSFEM) proposed by Ghanem and Spanos [6], for probabilistic representation of response quantities, also for reliability assessment. Although the SSFEM is well suited to second-moment analysis involving random fields, Sudret and Der Kiureghian [19], have shown that it has limited applicability to reliability problems involving small failure probabilities.

As an alternative, the PDF of the structural response can be obtained using MC simulation methods and traditional fitting techniques to estimate the parameters of an assumed theoretical probability distribution that would describe the randomness of the response in the limit state function. However the accuracy of the results depends on the adequacy of the selected probabilistic model to represent the variability of the response. The probabilistic models are chosen based on physical considerations or statistical evidences based on a large number of samples. Normal or other asymptotic distributions may be adopted.

A more efficient and elegant approach is to use a distribution free technique for estimating the PDF of the response such as the Maximum Entropy Method. The Maximum Entropy Method (MEM) was suggested by Jaynes [20], as a rational approach for estimating the probability density function of a random variable under specified moment constraints when little information is available on the data. The approach results in least biased probability distribution among all possible distributions that are consistent with available data. The Maximum Entropy Method is widely recognized as an efficient stochastic modelling tool when small number of samples are available, which has been successfully applied to many problems and in a wide variety of fields (e.g. [21–26]).

In this context an approach is proposed in this paper that combines the use of the Maximum Entropy Method for probabilistic modelling of the response with the FORM for reliability assessment. The probabilistic model of the response obtained by the MEM is compared with the 3-parameter lognormal model derived by fitting the distribution to particular fractiles of the response calculated by means of inverse FORM [27,8]. Then, the reliability analysis is performed using FORM with explicit limit state functions based on the different probabilistic models of the response. These results are also compared with the ones obtained using FORM directly linked to the FEA and the Response Surface Method (RSM). The proposed approach is first applied to a simple elastic problem of a portal frame and then applied to the analysis of a ship stiffened plate subjected to longitudinal compression and lateral pressure where the efficiency of the different approaches are compared.

2. Maximum entropy based method for reliability analysis

In most of applications, the probability density function of the response and probabilities associated with specific failure events are both of interest. Moreover, knowledge of the FORM-design point (i.e. the most probable point) is also relevant. This makes the Statistical Response Characterization Method (SRCM) attractive due to its simplicity when combined with efficient sampling and probabilistic modelling techniques.

In this paper, an improved method based on the concepts of the SRCM is proposed by using the Maximum Entropy Method (MEM) to obtain the probability distribution of the structural response. The reliability is then obtained by FORM/SORM or MC using an explicit limit state function of the type: