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Interaction diagrams for fire-exposed reinforced concrete sections

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A R T I C L E   I N F O

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A B S T R A C T

Fire safety of Reinforced Concrete (RC) columns is an important design aspect to ensure the overall integrity of structures during fire events. Currently, fire ratings of RC sections are achieved using prescriptive methods. As new codes are moving towards performance based design, practitioners are in need of rational design tools to assess the capacity of heated sections. To construct the axial force-moment interaction diagram of a RC section using existing numerical methods, high computation demand and knowledge of heat transfer and stress analysis are required. This paper presents the derivation of a set of formulas that can be used to estimate the average temperature distribution within the concrete section and the corresponding internal forces. The utilization of these formulas to construct interaction diagrams of fire-exposed RC sections is then explained. The proposed formulas are validated by comparing their predictions with experimental and analytical results by others.

1. Introduction

Ongoing development in material and construction technology challenges structural engineers to achieve innovative designs. One of the key issues that they need to address is the fire safety of the designed structure. For example, the evacuation process for high-rise buildings must be carefully considered and the structural safety of the building should be assured. When Reinforced Concrete (RC) columns experience elevated temperatures, their capacities are significantly reduced. This reduction is directly related to the deteriorated mechanical properties of concrete and steel [1]. Fire temperatures also induce nonlinear thermal and transient creep strains [2].

The most reliable approach to assess the fire endurance of RC columns is the experimental approach [3,4]. However, its cost and limitations make it unsuitable for regular design. The capacity of heated RC columns can be analytically assessed using axial force-moment interaction diagrams [5]. Available methods to construct these diagrams include the Eurocode 2 [6] and the Finite Element Method (FE). Eurocode 2 [6] recommends the use of the 500 °C isotherm method. This method assumes that concrete is either undamaged or fully-damaged depending on its temperature as compared to 500 °C. Ignoring concrete damage, when its temperature is lower than 500 °C, can result in unsafe predictions [5,8]. The FE method can be applied for a range of load eccentricities while varying the applied axial force until failure occurs. Such method needs to be repeated for different fire durations, which makes it computationally expensive [5,7]. El-Fitiany and Youssef [8–10] proposed a sectional analysis method that relies on converting the two-dimensional (2D) temperature distribution to an average one-dimensional (1D) temperature distribution to predict the uniaxial behavior of heated sections at different axial load levels (λ). This method can be used in similar manner to the FE method to construct the interaction diagrams. Although it requires much less computational effort, it is still considered unpractical for design engineers.

This paper proposes a practical approach to construct the interaction diagrams of RC sections during fire exposure. The proposed approach accounts for the effect of fire on concrete properties and strains. It also eliminates the need to divide the section into smaller elements. The following sections explain the development of an efficient method to calculate an average 1D temperature distribution, the derivation of closed form solutions for concrete internal stresses, and a simplified method to construct the interaction diagrams for fire-exposed columns.

2. Interaction diagrams using sectional analysis

Fire temperature decreases concrete and steel mechanical properties and induces thermal and transient strains. A sectional analysis approach suitable for the analysis of rectangular RC sections at elevated temperatures was proposed by El-Fitiany and Youssef [8,9]. This approach was validated by comparing its results with experimental and analytical work conducted by others. The use

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Nomenclature

A_1	factor used in calculating internal concrete force, equals to $e^{y_1\ z_2}$
A_2	factor used in calculating internal concrete force, equals to $e^{y_2\ z_2}$
b	column width in x direction
C_c	internal compression force in concrete
$C_{c\ o(v)}$	concrete compression force at $\varepsilon_{cT} \leqslant (\varepsilon_{oT} + \varepsilon_{tr})$ for variable T_{avg} distribution
$C_{c\ o(v)} \cdot y$	concrete moment about x axis at $\varepsilon_{cT} \leqslant (\varepsilon_{oT} + \varepsilon_{tr})$ for variable T_{avg} distribution
$C_{c\ u(v)}$	concrete compression forces at $\varepsilon_{cT} > (\varepsilon_{oT} + \varepsilon_{tr})$ for variable T_{avg} distribution
$C_{c\ u(v)} \cdot y$	concrete moment about x axis at $\varepsilon_{cT} > (\varepsilon_{oT} + \varepsilon_{tr})$ for variable T_{avg} distribution
$C_{c\ o(c)}$	concrete compression force corresponding to $\varepsilon_{cT} \leqslant (\varepsilon_{oT} + \varepsilon_{tr})$ for constant T_{avg}
$C_{c\ o(c)} \cdot y$	concrete moment about x axis at $\varepsilon_{cT} \leqslant (\varepsilon_{oT} + \varepsilon_{tr})$ for constant T_{avg}
$C_{c\ u(c)}$	concrete compression force corresponding to $\varepsilon_{cT} > (\varepsilon_{oT} + \varepsilon_{tr})$ for constant T_{avg}
$C_{c\ u(c)} \cdot y$	concrete moment about x axis at $\varepsilon_{cT} > (\varepsilon_{oT} + \varepsilon_{tr})$ for constant T_{avg}
f'_c	compressive strength for concrete at ambient temperature
f_y	yield strength of steel bars at ambient temperature
f'_{cT}	reduced compressive strength at elevated temperatures
f_{cT}	compression stress in heated concrete
f_{yT}	reduced yield strength of reinforcing bars at elevated temperatures
f_{sT}	compression or tension stress in heated steel bars
$(f_{cT})_{avg}$	average concrete compressive stresses
h	cross-section height
M	flexural moment
n_w	ratio between the surface temperature and the fire temperature
n_x and n_y	ratios between the internal and surface temperatures due to heating in the x and y directions, respectively
P	axial load
r	length of descending branch in concrete stress-strain relationship
t	fire duration
t^*	Equivalent fire duration assuming ISO 834 standard fire
T	temperature in degree Celsius [1 °F = 1.8 °C + 32]
T_σ	temperature produces the same average concrete strength for the layer
T_{th}	algebraic average temperature of the elements within each layer
T_{xy}	temperature rise at any point located at (x,y)
T_{avg}	algebraic average distribution along the section height
$T_{avg\ 1}$	average temperature for regions affected by heating from either left or right
$T_{avg\ 2}$	average temperature for regions not affected by heating from left or right
$T_{avg\ 3}$	average temperature due to heating from the left and right sides simultaneously
T_f	fire temperature
$T_{f(ISO)}$	ISO 834 standard fire temperature at a modified fire duration t^*
x, y	horizontal and vertical coordinates for any point within the column/beam section, origin located at bottom left of the section
y_1, y_2	boundaries of internal concrete compression force measured in y direction
z	boundary of fire affected regions
z_1, z_2	constants of average temperature fitting equation, Eq. (7)
z_3, z_4	constants defining the linear variation of ε_{cT} in y direction, Eq. (15)
ε	total concrete strain at elevated temperatures
ε_{th}	unrestrained thermal strain of concrete
ε_{tr}	transient creep strain in concrete
ε_c	instantaneous stress-related strain
ε_{cT}	equivalent mechanical strain in concrete during fire exposure
$\overline{\varepsilon_{th}}$	equivalent linear thermal strain
ε_i	unrestrained thermal axial strain
ε_{st}	self induced thermal strains
ε_{sT}	equivalent mechanical strain in steel during fire exposure
ε_o	strain at maximum stress of unconfined concrete at ambient temperature
ε_{oT}	value of ε_c at peak stress f'_{cT}
ε_{uT}	ultimate compressive strain of concrete, Eq. (10)
$\varepsilon_{cT\ max}$	compression strain corresponding to the flexural capacity
$\Delta\varepsilon$	difference between ε_{uT} and $(\varepsilon_{oT} + \varepsilon_{tr})$ equals to 0.02
ψ_i	unrestrained thermal curvature
λ	axial or flexural load level
ρ	reinforcement ratio
Γ	compartment time factor

of this approach to evaluate the interaction diagrams for a fire-exposed RC section involves the following steps:

1. At specific fire duration, the section is divided into a number of elements, Fig. 1a, and the temperature distribution is predicted using the Finite Difference Method (FDM) [1].
2. The heat transfer elements are grouped into horizontal layers to conduct sectional analysis. To accurately predict the section behavior, two average temperatures, T_σ and T_{avg}, are calculated for each layer. T_σ represents the temperature corresponding to the average concrete strength for the layer. T_{avg} represents the algebraic average temperature of the elements within each layer and is suitable to calculate thermal and transient creep strains [9].
3. The total concrete strain at elevated temperatures (ε) is composed of three terms: unrestrained thermal strain (ε_{th}), instantaneous stress related strain (ε_c), and transient creep strain (ε_{tr}). The total strain is given by Eq. (1).

$$\varepsilon = \varepsilon_{th} + \varepsilon_c + \varepsilon_{tr} \tag{1}$$

The nonlinear thermal strain (ε_{th}) distribution, Fig. 1f, is calculated using T_{avg}. The thermal strains of the steel bars are calculated based on the concrete temperature at their locations. (ε_{th}) is then converted to an equivalent linear thermal strain ($\overline{\varepsilon_{th}}$), Fig. 1c, by considering self-equilibrium of internal thermal forces in concrete and steel layers. ($\overline{\varepsilon_{th}}$) is represented by the value of the axial strain (ε_i). The corresponding curvature is equal to zero as the section is assumed to be heated from four faces. Fig. 1e shows the differences between the equivalent linear and nonlinear thermal strains, which represent the self-induced thermal strains (ε_{st}). These strains are assigned as initial strains for the concrete and steel layers to model the corresponding self-induced self-equilibrating thermal stresses. The terms ε_{st}, ε_c, and ε_{tr} are lumped into an equivalent mechanical strain as shown by Eq. (2).

$$\varepsilon = \overline{\varepsilon_{th}} + (\varepsilon_{st} + \varepsilon_c + \varepsilon_{tr}) = \overline{\varepsilon_{th}} + \varepsilon_{cT} \tag{2}$$