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Shear behaviour of non-prismatic steel reinforced concrete beams

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ABSTRACT

Large reductions in embodied carbon can be achieved through the optimisation of concrete structures. Such structures tend to vary in depth along their length, creating new challenges for shear design. To address this challenge, nineteen tests on non-prismatic steel reinforced concrete beams designed using three different approaches were undertaken at the University of Bath. The results show that the assumptions of some design codes can result in unconservative shear design for non-prismatic sections.

1. Introduction

Non-prismatic concrete beams can provide steel and concrete savings when used to replace equivalent strength prismatic elements. In a variable section reinforced concrete beam a portion of the shear force may theoretically be carried by a suitably inclined top or bottom flange, yet such beams have been found to fail prematurely, suggesting codified methods are unable to account for the varying section shapes found in optimised structures. Given that optimised structures tend to be non-prismatic, understanding these failures and providing appropriate guidance for their design is hugely important.

2. Tapered beams

2.1. Shear behaviour

The derivation of shear stresses through equilibrium considerations of a homogenous uncracked and isotropic beam is relatively straightforward, but the behaviour of a reinforced concrete section is more complex. In a reinforced section, cracks will form when the principal tensile strain exceeds the tensile capacity of concrete and these diagonal cracks typically propagate from the tension face of the member towards the neutral axis.

There are conventionally considered to be six contributing factors by which a reinforced concrete beam can carry shear, Fig. 1. When present, shear reinforcement carries stress over cracks as they open under loading and confines the section. Although aggregate interlock is estimated to carry significant shear force in the uncracked section [1], as cracks open the capacity to transfer stresses via aggregate interlock is minimal [2]. Dowel action by longitudinal reinforcement is contentious, with Kotsovos [2] showing it to be extremely limited in the prismatic section.

The behaviour of prismatic and tapered concrete sections in shear is compared in Fig. 1. In sections that taper towards their supports, the interaction of the diagonal cracks with the path of the compression force at the supports is assumed to be critical.

Inclined compression or tension forces can theoretically affect the shear resistance of the section. It is suggested [3–7] that for sections whose depth increases in the direction of increasing moment an effective shear force for design be given by Eq. (1), which is valid for members with shear reinforcement:

$$V'_{Ed} = V_{Ed} - V_{ccd} - V_{td} \tag{1}$$

where V'_{Ed} is the reduced design shear force; V_{Ed} is the shear force on the cross section; V_{ccd} is the vertical component of force in the inclined compression chord and V_{td} is the vertical component of the inclined tension chord. M_{Ed} is the moment on the cross section.

Provided suitable limits on stress in the web compression strut are not exceeded, the sum of V_{ccd} and V_{td} (Eq. (1)) could theoretically be made equal to the applied shear force, negating requirements for transverse steel. For a beam with yielding tension reinforcement of constant area (A_s) and without normal force this could be achieved by placing the bar at an effective depth that is proportional to the bending moment at each point along the beam

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Nomenclature

A_{sw} area of transverse reinforcement (mm^2)
a_{vx} M_{ax}/V_{ax} (mm) in the CFP method
b_w web width (mm)
dx an increment of length
f_{yk} characteristic yield strength of steel reinforcement
M_{ax} applied bending moment (N mm) on a section in the CFP method
M_{cx} is the moment corresponding to shear failure (N mm) in the CFP method
$M_{Ed,i}$ the applied moment at position i
M_f the flexural capacity (N mm) in the CFP method
s transverse reinforcement spacing (mm)
V'_{Ed} the reduced design shear force, Eq. (1)
V_{ax} applied shear force (N) on a section in the CFP method
V_{cx} shear force at failure in the CFP method
$V_{Ed,i}$ the applied shear force at position i
V_f shear force corresponding to flexural failure in the CFP method
V_{td} vertical component of force in the bar
$V_{td,i}$ the vertical component of force in the bar at position i
x (subscript) denotes a given cross-section at a distance x mm from the support in the CFP method
z_i the lever arm between tension and compression forces at position i
ρ_w ratio of the area of tension steel to the web area of concrete to the effective depth in the CFP method
ρ_w $A_{sw}/s/b_w$ (reported in %)

(Eq. (2)). Such an approach would make the vertical component of force in the bar (V_{td}) theoretically equal to the applied shear force, Eq. (3).

$$z_i = \frac{M_{Ed,i}}{A_s f_{yk}} \tag{2}$$

$$V_{Ed,i} = \frac{dM_{Ed,i}}{dx} = V_{td,i} \tag{3}$$

where z_i is the lever arm between tension and compression forces at position i; $M_{Ed,i}$ is the applied moment at position i; A_s is the constant area of longitudinal reinforcement and f_{yk} is its characteristic yield strength; $V_{Ed,i}$ is the applied shear force at position i; dx is an increment of length and $V_{td,i}$ is the vertical component of force in the bar at position i.

However, utilising a longitudinal bar to provide vertical force capacity close to the supports in a simply supported beam requires the bar to be fully anchored at its ends and yielded along its entire length. Furthermore, for a structure subject to an envelope of loads the longitudinal reinforcement position will be determined by the maximum moment on each section. It is feasible that the maximum moment and shear forces on a section will not originate from the same load case. In such a situation, a bar placed for moment capacity will then be incorrectly inclined to provide the desired vertical force, and thus additional transverse reinforcement will be required.

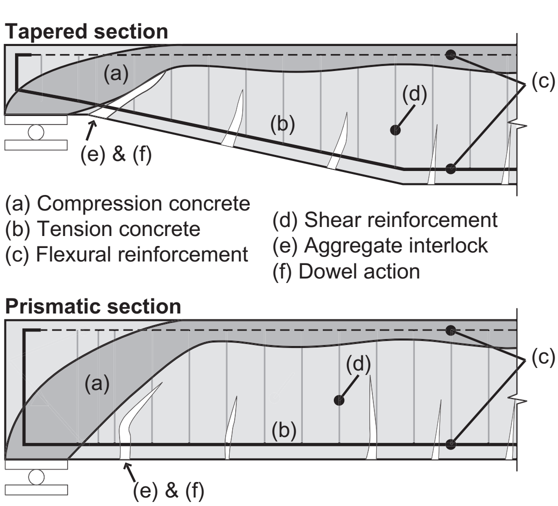

Fig. 1. Contributing factors to shear resistance in tapered and prismatic sections.

2.2. Design methods

2.2.1. Truss model

ACI 318 [8] and BS EN 1992-1-1 [4] allow the shear capacity of a tapered section with transverse reinforcement to include the effects of inclined tension and compression forces, Eq. (1). Both codes are based on the truss analogy, the premise of which [9,10] is that cracked concrete in the web resists shear by a diagonal uniaxial compressive stress in a concrete strut, pushing the flanges apart and causing tension in the stirrups that are then responsible for holding the section together. With a compression strut angle of 45°, the model consistently underestimates shear strength. To correct this ACI 318 [8] adds a 'concrete contribution', while BS EN 1992-1-1 [4] assumes that once cracked the concrete provides no contribution to shear capacity and instead allows a flatter strut angle (down to 22°, subject to stress limits in the diagonal concrete strut) to be chosen, with both approaches replicating experimental observations.

The additional tensile force, ΔF, arising from the normal stress components of the inclined web compression struts of the truss model [4,8] must be included when calculating the force in the inclined chords to prevent over-estimation of the contribution of an inclined chord to shear capacity.

2.2.2. Compressive force path method

The compressive force path (CFP) method premises that the behaviour of a reinforced concrete beam can be simplified into three elements – a concrete frame, a steel tie of flexural reinforcement and a zone of concrete cantilevering teeth which form between successive cracks in the concrete section [11], Fig. 2. The uncracked compression zone is proposed to sustain both the compressive flexural force and the entire shear force. Since

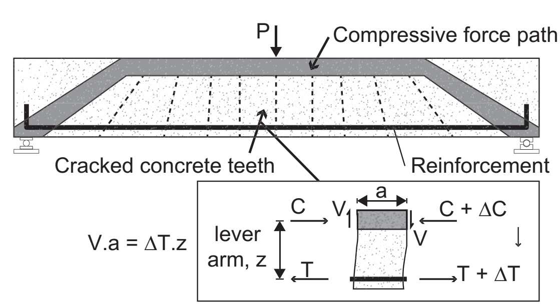

Fig. 2. The CFP method [11].