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A generic time domain implementation scheme for non-classical convolution damping models

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ABSTRACT

A generic time domain integration formulation for linear systems with non-classical convolution damping models is presented. The non-classical damping force is assumed to depend on the past history of velocity through a convolution integral over a causal dissipative kernel function. The time domain implementation formulation is developed using the Newmark constant average acceleration framework. To emphasize the accuracy of the proposed scheme, numerical comparisons are made for a three-degrees-of-freedom system and an axially vibrating rod problem reported in literature. The generality of the formulation is shown by simulating the response of a cantilever beam enhanced with two known standard dissipation functions: the exponential and the Gaussian model. The implementation of the proposed scheme is also presented.

1. Introduction

Structural dynamic analysis is mainly characterized by three forces: the inertia force, the damping force and the stiffness force. Out of these, the mechanics for both the inertia and the stiffness force are well understood whereas the damping force represents an observed phenomenon. Damping, in simplistic terms, could be defined as the process by which a certain portion of the energy in a vibrating system is irreversibly lost causing decay in the system response. Despite having a large amount of literature on the subject, the underlying physics is only known in a phenomenological ad hoc manner, making damping an overall mystery in the general dynamic analysis of structures. A major reason of this could be the fact that there is no single universally accepted model for damping [1]. The ambiguity involved in the modelling of damping is mainly due to the intricacies involved in understanding the *state variables* controlling the damping forces [2].

In classical dynamics, for a discrete system, the damping force is predominantly represented by a viscous model, proposed by Rayleigh in 1877 [3], through his famous dissipation function. This is the most popular model currently used both in practice and in research mainly due to its simplicity, because the whole phenomenon of damping is mathematically reduced to the estimation of a single parameter called damping ratio [4]. Many studies in the past have shown that the viscous damping model suggested by Rayleigh is only a mathematical idealization and the “real damping” could be different [1,2]. This sort of mathematical idealization may lead to “modelling errors” in dynamic response analysis. The manifestation of these modelling errors has been obtained in various branches of engineering [5,6]. This has paved the way for an increasing interest in other types of models which represent damping forces in a more general manner as compared to the classical viscous damping model [7].

One such model of great interest is the non-classical convolution damping model in which the damping force is represented by convolution integrals, which take into account the complete past history of responses other than just the instantaneous velocities as represented by viscous damping model [2,7]. The damping force $f(t)$, using such a model could be expressed as,

$$f(t) = \int_0^t C_k g(t-\tau)\dot{u}(\tau)d\tau \tag{1}$$

In this equation, $g(t)$ represents the damping kernel function and C_k represents the damping coefficient. The kernel function $g(t)$ could represent any causal model which makes the energy dissipation functional non-negative [8]. Normally, $g(t)$ is taken as the normalized damping function [9], which satisfies,

$$\int_0^\infty g(t)dt = 1.0 \tag{2}$$

In literature this is commonly referred to as non-viscous damping model [1], considering the fact that integration by parts of Eq. (1) would result in the damping force being expressed as a function

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of displacement. But considering the fact that damping force in its form as given in Eq. (1) is in a convolution format depending on the past history of velocities, the authors prefer to address the formulation as non-classical convolution damping.

Incorporation of this model described in Eq. (1) into the dynamic equilibrium equation would result in an integro-differential equation expressed as,

$$M\{\ddot{u}(t)\} + \int_0^t C_k g(t-\tau)\{\dot{u}(\tau)\}d\tau + K\{u(t)\} = \{P(t)\} \tag{3}$$

Here, M and $K \in R^{N\times N}$ are the mass and stiffness matrices and $\{P(t)\} \in R^N$ is the force vector and the convolution damping term has the same descriptions as defined above in Eq. (1) with C_k representing the damping coefficient matrix. Similarly $\{\ddot{u}(t)\}, \{\dot{u}(\tau)\}$ & $\{u(t)\}$ are the acceleration, velocity and displacement vectors. The initial conditions associated with Eq. (3) are as follows,

$$\{u(0)\} = \{u_0\} \in R^N;\ \text{and}\ \{\dot{u}(0)\} = \{\dot{u}_0\} \in R^N \tag{4}$$

As Eq. (3) is an integro-differential equation, no classical methods like Newmark family methods can be applied directly for its solution [9]. So in recent years, considerable amount of research efforts have been put to solve Eq. (3) to obtain system responses. The majority of the research conducted presented solution schemes for systems in which the damping kernel function adopts an exponential model. McTavish and Hughes [10] adopted the double exponential model proposed by Golla and Hughes [5] for the damping kernel function and proposed a scheme which resulted in a second order equation of motion. This second order equation of motion was then solved using classical time integration techniques. This scheme is known as the GHM method. Its main drawback is the use of a large number of internal dissipation coordinates used to capture the frequency dependent viscoelastic behavior which enlarges the matrix size. Adhikari and Wagner [7] proposed a time domain analysis scheme for systems with exponential damping kernel function based on an extended state space representation of the equation of motion. The efficiency of the proposed numerical method for the calculation of displacements relies on the elimination of the need for explicit calculation of the velocities and usually a large number of internal variables at each time step. Cortes et al. [9] employed Laplace transformation on the equation of motion containing exponential damping kernel function and derived an equivalent second order equation of motion which was then solved using implicit standard time integration schemes. The main advantage of this method was that it did not employ any internal variables which normally increased the size of the problem. The main disadvantage is that the Laplace transformation results in a differential equation with time derivative orders higher than two and the authors admit the difficulty in performing the mathematical manipulation, when the damping model has more than two exponential kernel functions.

In this paper, a generic time domain formulation for multi-degree of freedom systems represented by Eq. (3) is presented. This is called 'generic' because in comparison to majority of the earlier works, this formulation could be used for any causal model that the damping kernel function adopts. The other main advantage is that the formulation uses the Newmark framework with some modifications and as a result could be easily incorporated into an existing commercial software package. This aspect has been demonstrated by incorporating the proposed formulation into the commercial package "Ruaumoko" maintained by the second author and is currently under testing. Latest version of Ruaumoko yet to be released gives the option of incorporating convolution damping models for dynamic analysis of structures, though presently it is fully restricted to linear analysis. The implementation logic is presented in Appendix B.

2. Mathematical derivation

In this section the time domain formulation is developed using Newmark constant average acceleration frame work. At time t, the dynamic equilibrium equation with linear generic damping model of the form presented in Eq. (1) is given by Eq. (3).

At time $t+\Delta T$, this equation becomes,

$$M\{\ddot{u}(t+\Delta T)\} + \int_0^{t+\Delta T} C_k g(t+\Delta T-\tau)\{\dot{u}(\tau)\}d\tau + K\{u(t+\Delta T)\} = \{P(t+\Delta T)\} \tag{5}$$

Here, a revised Newmark constant average acceleration method (rather than the classical incremental approach) is adopted to solve the equation of equilibrium at time $t+\Delta T$ [11]. The fundamental assumption in the classical Newmark constant average acceleration method is that the acceleration is assumed to be constant during the time step with a value equal to the average of the accelerations at the beginning and end of the time step. The classical Newmark method starts with the difference in the response between two successive time steps ΔT apart and results in solving an incremental equilibrium equation. But the revised Newmark scheme starts with equation of equilibrium at time $t+\Delta T$ (refer Appendix A for further details).

In order to have a convenient formulation for the Newmark implementation, the convolution integral is split and is given as follows;

$$M\{\ddot{u}(t+\Delta T)\} + \int_0^{t} C_k g(t+\Delta T-\tau)\{\dot{u}(\tau)\}d\tau + \int_t^{t+\Delta T} C_k g(t+\Delta T-\tau)\{\dot{u}(\tau)\}d\tau + K\{u(t+\Delta T)\} = \{P(t+\Delta T)\} \tag{6}$$

The above equation can be rewritten in incremental terms as (refer Appendix A),

$$M\{\ddot{u}(t)+\Delta\ddot{u}\} + \int_0^{t} C_k g(t+\Delta T-\tau)\{\dot{u}(\tau)\}d\tau + \int_t^{t+\Delta T} C_k g(t+\Delta T-\tau)\{\dot{u}(\tau)\}d\tau + K_S\{u(t)\} + K_T\{\Delta u\} = \{P(t+\Delta T)\} \tag{7}$$

Here, $\{\Delta u\}$ refers to increment in displacement, $\{\Delta\ddot{u}\}$ refers to increment in acceleration, K_T refers to the tangent stiffness and K_S refers to the secant stiffness. For linearly elastic structures, the secant and tangent stiffness matrices are identical to the initial elastic matrices. Though the present paper addresses only the linear dynamics scenario, the above notation of secant and tangent stiffness matrix is mainly retained to keep the generality of the Newmark integration scheme. In Eq. (7), both the acceleration and displacement within a time step are represented in their incremental components, whereas the velocity still remains as a continuous function. There are two integral terms containing velocity functions in Eq. (7); one varies from 0 to t and the other from t to $t+\Delta T$. So, the velocity term varies both globally (i.e. 0 to t) and locally (i.e. t to $t+\Delta T$). Now for convenience, let's denote the convolution integral from 0 to t as,

$$\{F_{damp}\} = \int_0^{t} C_k g(t+\Delta T-\tau)\{\dot{u}(\tau)\}d\tau \tag{8a}$$

So Eq. (7) could be rewritten as,

$$M\{\ddot{u}(t)+\Delta\ddot{u}\} + \{F_{damp}\} + \int_t^{t+\Delta T} C_k g(t+\Delta T-\tau)\{\dot{u}(\tau)\}d\tau + K_S\{u(t)\} + K_T\{\Delta u\} = \{P(t+\Delta T)\} \tag{8b}$$