Engineering Structures 69 (2014) 149–157

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Biaxial capacity of rigid footings: Simple closed-form equations and experimental results

J. Paul Smith-Pardo [a,*], Albert Ortiz [b,1], Carlos A. Blandon [c,2]

[a] Department of Civil and Environmental Engineering, Seattle University, Seattle, WA, USA
[b] Department of Civil and Environmental Engineering, University of South Carolina, Columbia, SC, USA
[c] Department of Civil Engineering, Escuela de Ingeniería de Antioquia, Medellin, Colombia

ARTICLE INFO

Article history:
Received 19 June 2013
Revised 21 December 2013
Accepted 11 March 2014
Available online 12 April 2014

Keywords:
Biaxial capacity of foundations
Limit analysis
Performance-based design
Experimental results
Design-aids

ABSTRACT

This manuscript first presents an analytical procedure to derive dimensionless charts for the analysis and design of rigid rectangular foundations under axial load and biaxial moment. It then shows that conditions of symmetry in the normalized domain of the problem lead to practical closed-form equations that provide the entire coupled axial-load-biaxial-bending capacity envelope of shallow rectangular footings. The resulting relations may find direct application in the performance-based seismic analysis/design of soil-structure systems for which the foundations are vulnerable (i.e., prone to uplifting and/or soil plastification). It is demonstrated that the equations are a generalized version of the widely used equivalent width concept proposed by Meyerhof. Results from an experimental program involving 19 tests with 200 mm square and 200 × 300 mm rectangular foundations models under biaxial loading are presented to show that the proposed simple equations provide reasonable estimates of the measured capacity. Further comparisons with large-scale and small-scale foundation models available in the literature suggest that, similar to Meyerhof's equivalent width concept, the proposed formulation is relatively independent of scale effects.

1. Introduction

In the context of seismic analysis and design, performance-based methodologies require the estimation of the force and displacement capacity of structural members in the main lateral load resisting system of the structure under evaluation. In order to perform fully consistent soil-structure interaction analyses, the designer/analyst should also estimate the axial-load-biaxial-bending capacity envelope of the foundations. This is particularly important when a building, deemed to be inadequate by current seismic code design provisions, is rehabilitated through the installation of shear walls and major modifications of the footings are not possible. In this case the original foundations may be weaker than the new supported members and thus vulnerable to uplifting and/or yielding of the supporting soil under a major seismic event [1]. Even in the absence of vulnerable foundations, seismic rehabilitation standards [2] typically require consideration of the interaction between the structure and the supporting soil for buildings located in moderate and high seismic risk zones.

The condition of vulnerable foundations can sometimes be advantageous to the seismic performance of structures because of the energy dissipation due to yielding of the soil and period shifting associated to rocking and uplifting [3–6]. Recently, Deng [7] and Deng et al. [8,9] have shown that nonlinear soil-structure interaction effects can in fact reduce residual structural rotations, displacement demand on the structural components, and the collapse potential of structures [7–9]. Anastasopoulos et al. [10] presented a seismic design approach which takes advantage of soil yielding to protect the superstructure. Despite the evidence, current codes [11,12] discourage engineers from relying on the energy dissipation at the foundation/soil interface because of the potential for higher drift rations and damage to nonstructural elements. Therefore, under such provisions, the proper estimation of the ultimate capacity of foundations becomes even more important in order to determine whether soil yielding may occur under a major seismic event.

Footings that are subjected to axial load and large bending moments may have portions that do not act in bearing. In order to analyze or design for such a loading condition, the engineer must compute the bearing pressures and the location of the neutral axis.

* Corresponding author. Address: 901 12th Ave., Seattle, WA 98122-1090, USA. Tel.: +1 206 296 5901; fax: +1 206 296 2173.
E-mail address: smithjh@seattleu.edu (J. P. Smith-Pardo).
[1] Tel.: +57 4 340 5231.
[2] Tel.: +57 4 354 9090.

http://dx.doi.org/10.1016/j.engstruct.2014.03.007
0141-0296/

For footings subject to biaxial bending, the solution of this problem can be tedious and amenable only to numerical approaches. Fortunately, the calculation of the vertical load capacity of shallow footings with eccentricities in two directions has been extensively studied over the years [13–18]. Yamamoto et al. [19] developed an analytical solution applicable to shallow foundation on compressible sand. Rayhani and ElNaggar [20], on the other hand, developed a numerical nonlinear elastic-plastic model that simulates the seismic behavior of a rectangular building on soft clay. Bouassida et al. [21] studied the bearing capacity of a rigid foundation on a soil reinforced by a group of floating columns using limit analysis. More recently Rodriguez-Gutierrez and Aristizabal-Ochoa [22,23] developed an analytical method to calculate the capacity of a rigid footing of any shape subjected to an eccentric axial load causing biaxial bending; the authors considered three different types of pressure distribution at the soil-footing interface (uniform, linear, and parabolic) and developed non-dimensional nomograms for the analysis and design of rigid footings of different shapes.

A common limitation of all the studies listed above is the lack of experimental results to support the proposed analytical or numerical solutions. In addition, the design aids and numerical solutions are not closed-form so they do not lend themselves as tools that practitioner engineers may readily use to conduct nonlinear soil-structure interaction analyses in a systematic manner.

This paper intends to alleviate those limitations by: (1) presenting an analytical solution and develop non-dimensional charts that can be used to estimate the biaxial capacity of rectangular footings; (2) recognizing symmetry conditions in the normalized domain of the problem to derive approximate closed-form equations for the calculation of the entire axial-load-biaxial-bending (PMM) capacity surface, (3) demonstrating that the closed-form equations become a generalization of the widely accepted equivalent width concept proposed by Meyerhof [24]; and (4) presenting the results of an experimental program with nineteen 200 × 200 mm and 200 × 300 mm footing models under biaxial bending and comparing the measured capacity with the values obtained using the proposed closed-form equations.

2. Uniaxial capacity of foundations

As depicted in Fig. 1, the eccentric load bearing capacity, P_n, of a shallow foundation can be related to its concentric load capacity, P_0, by assuming that in both cases the soil can fully plastify at the same stress level σ_0. Conditions of equilibrium in the two configurations lead to the well-known equivalent width concept proposed by Meyerhof [24]:

$$\frac{P_n}{P_0} = \left[1 - \frac{2E}{B}\right] \tag{1}$$

where E is the uniaxial eccentricity of the load and B is the foundation size in the plane of bending.

The equivalent width concept, admittedly, does not strictly obey principles of mechanics but it is rather a convenient yet useful tool that imposes a uniform contact stress distribution in order to facilitate the design of foundations. A coupled axial-load-bending capacity (PM) envelope can be established by setting $M_n = P_nE$ in Eq. (1) and rearranging:

$$\frac{M_n}{P_0B} = \frac{1}{2}\frac{P_n}{P_0}\left[1 - \frac{P_n}{P_0}\right] \tag{2}$$

The practical relevance of this simple relation for seismic design should be emphasized; if an engineering practitioner desires to design a capacity-protected shallow foundation, he/she would need to ensure that the axial load-moment interaction envelope at the base of the supported column or wall falls inside of the M–P envelope given by Eq. (2).

3. Derivation of design aids for biaxial capacity of foundations

An extension of Meyerhof's assumption is shown in Fig. 2 for the plastic limit state of a rigid footing with plan dimensions B_X and B_Y subjected to combined ultimate axial load P_n and moments M_{nX} and M_{nY} with respect to the centroidal X and Y axes.

For the derivation, it is convenient to introduce a statically equivalent system with eccentricities $E_X = M_{nY}/P_n$ and $E_Y = M_{nX}/P_n$ as shown in Fig. 2b. The estimation of the bi-eccentric load capacity P_n requires finding the location of the plastic neutral axis shown in Fig. 3 such that equilibrium of the footing is satisfied. This implies that the centroid of the shaded area, A, should be located at distances E_X and E_Y with respect to the centroid of the foundation so that the resultant of the plastic stress distribution (σ_0) has the same line of action of P_n.

The process can become tedious because the calculation of the plastic neutral axis would be required for each eccentricity pair and for every footing size. However, in order to obtain a solution that is independent of the footing size, the normalized problem shown in Fig. 4 can be solved instead. In this case, a normalized coordinate system is first defined as follows:

$$x = \frac{X}{B_X} \tag{3}$$

$$y = \frac{Y}{B_Y} \tag{4}$$

In this normalized domain, eccentricities are given by:

$$e_X = \frac{E_X}{B_X} \tag{5}$$

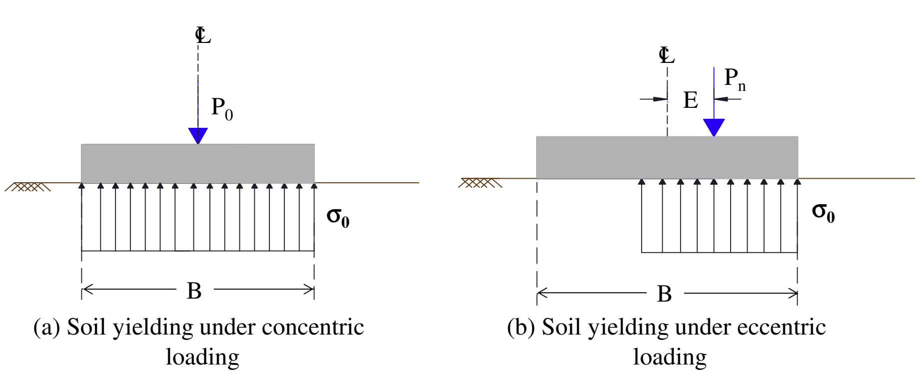

Fig. 1. Plastic limit state for shallow footings under concentric and eccentric loading.