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Minimum volume design of structures with constraints on ductility and stability

Luigi Palizzolo *, Alessandra Caffarelli [1], Pietro Tabbuso [2]

University of Palermo, Department of Civil, Environmental, Aerospace, Materials Engineering, DICAM, Viale delle Scienze, 90128 Palermo, Italy

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ABSTRACT

A minimum volume design problem of elastic perfectly plastic frame structures subjected to different combinations of fixed and seismic loads is presented, in which the design variables are considered as appertaining alternatively to a continuous assigned range as well as to appropriate discrete sets. The structure is designed so as to behave elastically for the applied fixed loads, to shakedown in presence of serviceability seismic conditions and to prevent the instantaneous collapse for suitably chosen combinations of fixed and high seismic loadings. In order to avoid further undesired collapse modes, the P-Delta effects are considered and the structure is also constrained to prevent element buckling. Furthermore, some suitable constraint on the structure ductility is imposed referring to the plastic strains generated during the transient phase structural response. The dynamic structural response is obtained by utilizing an appropriate modal technique referring to the response spectrum defined by the Italian code. The proposed minimum volume design problem is formulated, according to the required structural behaviour, on the ground of a statical approach. Different numerical applications related to steel frames conclude the paper.

1. Introduction

The greater part of the International codes related to the structural analysis and design prescribes that structures during their lifetime must satisfy different suitably defined requisites related to as many different potentially occurring conditions.

Such requisites can involve mechanical parameters, as for example in the case of the usual structural safety criteria, or they can regard kinematical aspects, being related to the different possible configurations that the structure can exhibit, limiting some suitable measures of the elastic and/or plastic deformation. Finally, further limits can be imposed on the structure behaviour preventing dangerous critical phenomena characterizing the structural response depending on the special structural typology, as for example the element buckling and the P-Delta effects.

As it is well known, the formulation of an analysis or a design problem requires the definition of appropriate models for the structure, for the material behaviour as well as for the acting loads. The classical definitions of models for structures and materials, here skipped for the sake of brevity, certainly guarantee a consistent and coherent description of the relevant phenomena from an engineering and a scientific point of view; on the contrary, the load model, even if deeply described, especially if some seismic action can occur, it is not usually able to represent the real load history; actually, the load history is "essentially" unknown and, as a consequence, the load model can just provide appropriate domains characterized by a very high probability of containing the effective load histories which will act on the structure.

An usual and reliable loading model for structures subjected to seismic actions is defined by considering appropriate combinations of fixed and perfect cyclic loads. In such a load condition, it is known that the structural response exhibits at first a transient phase in which the response does not possess any periodicity feature, and eventually a subsequent steady-state phase in which the structural response become cyclic with the same period as the applied loads.

The steady-state response can be considered as known, namely, it is possible to evaluate the maximum values of the structural response depending on the highest values of the loads appertaining to the assigned domains. In particular, if the structural behaviour remains elastic the determination of the response is definitely trivial; instead, if the acting loads cause some plastic strains, the (elastic or plastic) shakedown theory and the limit analysis allow

* Corresponding author. Tel.: +39 09123896752.
E-mail addresses: luigi.palizzolo@unipa.it (L. Palizzolo), alecaffa@msn.com (A. Caffarelli), pietro.tabbuso@unipa.it (P. Tabbuso).
[1] Tel.: +39 09123896752.
[2] Tel.: +39 0912389608.

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to determine a complete and reliable evaluation of the structural responses.

On the contrary, the structural response during the transient phase cannot be considered as known being unknown the real load history within the assigned relevant domain.

As a consequence, for a structure subjected to a load condition inside the (elastic or plastic) shakedown domain it is possible to verify the plastic admissibility in terms of resistances, being known the mechanical structural response, but it is not as much possible to verify prescribed limits on the structural displacements because the plastic deformations occurring during the transient phase are not known and cannot be computed. In this case it is just possible to effect some approximate evaluation of the chosen displacements making recourse to the so-called bounding techniques proposed in the field of the elastic shakedown theory and extended to the case of the plastic shakedown behaviour (see, e.g., [1–7]).

In the framework of the optimal design of elastic plastic structures (see, e.g., [8–20]), several authors devoted their studies proposing special formulation with constraints on chosen measures of displacements and/or strains and with further constraints on stability (see, e.g., [21–30]). These papers substantially concern design problem of structures described by continuous variables. Actually, it must be noticed that in practical engineering applications it should be better to formulate the relevant problem by making reference to discrete variables. On the other side, to the author's knowledge, the studies which treat formulations of the optimal design problem involving a discrete variable approach, are substantially devoted to the related computational aspects and deal with the proposing of special numerical algorithms (see, e.g., [31–37]).

As a consequence, the present paper aims to provide a wide and complete reference for the exposed topics. In particular, a compact formulation of the minimum volume design for elastic perfectly plastic steel frames is proposed; the structure's geometry is described by means of continuous and/or discrete design variables; the second order effects related to the so called P-Delta effects are approximately taken into account. Furthermore, the optimal structure must respect appropriate (mechanical) constraints related to different possible limit behaviours, further constraints related to the limitation of suitably chosen displacements occurring during the transient phase of the structural response are imposed and the element buckling is prevented. The seismic actions are described by making reference to the relevant spectra defined by the Italian code [38] and the elastic response to the dynamic loads is obtained by means of a classical modal technique.

In the author's opinion the proposed formulation guarantees a complete and coherent scientific development and it is able to represent an unitary approach for the several different specific problems of practical engineering interest prescribed by the analysis and design international codes.

Some numerical applications are effected; in particular, a flexural three floors elastic perfectly plastic plane steel frame is studied. The numerical results obtained by utilizing the proposed formulation allow us to deduce interesting remarks regarding the features of the continuous and the discrete design model and to evaluate the influence of the ductility constraint. Furthermore, the Bree diagram of the obtained designs are determined in order to investigate on the behavioural features of the relevant structures.

2. Fundamentals and structural model

In the present section some fundamentals related to the definition of appropriate models both for the frame structure and for the acting loads are introduced, together with some further remarks about the inelastic behaviour useful for the formulation of the relevant optimization problem.

2.1. The frame model

Let us refer to a typical plane frame constituted by n_b beam type elements, described by the Navier kinematical model, and by n_N nodes, each characterized by three degrees of freedom. Let us define, as usual, the following quantities:

- $\tilde{\boldsymbol{u}} = |\, \tilde{\boldsymbol{u}}_1 \quad \tilde{\boldsymbol{u}}_2 \quad \ldots \quad \tilde{\boldsymbol{u}}_{n_N} \,|$ as the vector collecting the displacements of the frame nodes (dimension $3 \cdot n_N$);
- $\widetilde{\boldsymbol{F}} = |\, \widetilde{\boldsymbol{F}}_1 \quad \widetilde{\boldsymbol{F}}_2 \quad \ldots \quad \widetilde{\boldsymbol{F}}_{n_N} \,|$ as the vector collecting the forces applied on the frame nodes (dimension $3 \cdot n_N$);
- $\tilde{\boldsymbol{d}} = |\, \tilde{\boldsymbol{d}}_1 \quad \tilde{\boldsymbol{d}}_2 \quad \ldots \quad \tilde{\boldsymbol{d}}_{n_b} \,|$ as the vector collecting the displacements of the element extremes (dimension $6 \cdot n_b$);
- $\widetilde{\boldsymbol{Q}} = |\, \widetilde{\boldsymbol{Q}}_1 \quad \widetilde{\boldsymbol{Q}}_2 \quad \ldots \quad \widetilde{\boldsymbol{Q}}_{n_b} \,|$ as the vector collecting the generalized stresses evaluated at the extremes of the elements (dimension $6 \cdot n_b$);
- $\widetilde{\boldsymbol{Q}}^* = |\, \widetilde{\boldsymbol{Q}}_1^* \quad \widetilde{\boldsymbol{Q}}_2^* \quad \ldots \quad \widetilde{\boldsymbol{Q}}_{n_b}^* \,|$ as the vector collecting the perfectly clamped element generalized stresses (dimension $6 \cdot n_b$);

where the over tilde means the transpose of the relevant quantity.

The static linear elastic analysis problem for the plane frame can be given in the following form:

$$\boldsymbol{d} = \boldsymbol{C}\,\boldsymbol{u}, \tag{1a}$$

$$\boldsymbol{Q} = \boldsymbol{D}\,\boldsymbol{d} + \boldsymbol{Q}^*, \tag{1b}$$

$$\widetilde{\boldsymbol{C}}\,\boldsymbol{Q} = \boldsymbol{F}. \tag{1c}$$

In Eq. (1) $\boldsymbol{C}$ is the compatibility matrix (dimensions $6 \cdot n_b \times 3 \cdot n_N$), $\boldsymbol{D}$ is a square block diagonal matrix (order $6 \cdot n_b$), each block representing the typical frame element stiffness, and $\widetilde{\boldsymbol{C}}$ is the equilibrium matrix. The solution to Eq. (1) is given in terms of displacements of the structure nodes and generalized stresses at the extremes of the frame beam elements:

$$\boldsymbol{u} = \boldsymbol{K}^{-1}\,\boldsymbol{F}^*, \tag{2a}$$

$$\boldsymbol{Q} = \boldsymbol{D}\,\boldsymbol{C}\boldsymbol{u} + \boldsymbol{Q}^* = \boldsymbol{D}\,\boldsymbol{C}\boldsymbol{K}^{-1}\boldsymbol{F}^* + \boldsymbol{Q}^*. \tag{2b}$$

In Eq. (2) $\boldsymbol{K} = \widetilde{C}DC$ is the frame elastic stiffness matrix (order $3 \cdot n_N$) and $\boldsymbol{F}^* = \boldsymbol{F} - \widetilde{C}Q^*$ is the equivalent nodal force vector.

2.2. Seismic action structural model and related load combinations

Let us make now reference to seismic actions and let us study the frame subjected to an horizontal ground acceleration $a_g(t)$. The chosen model is characterized by masses concentrated at each node so that the structure is a Multi-Degree-Of-Freedom (MDOF) one. The dynamic equilibrium equations reads:

$$\boldsymbol{M}\,\ddot{\boldsymbol{u}}(t) + \boldsymbol{B}\,\dot{\boldsymbol{u}}(t) + \boldsymbol{K}\,\boldsymbol{u}(t) = \boldsymbol{f}_G(t), \tag{3}$$

with $\boldsymbol{f}_G(t) = -\boldsymbol{M}\boldsymbol{\tau}a_g(t)$. $\boldsymbol{M}$ is the mass matrix and $\boldsymbol{B}$ the damping one (both of order $3 \cdot n_N$). $\boldsymbol{\tau}$ is the so-called influence vector (dimension $3 \cdot n_N$). Furthermore, the over dot means time derivative, so that $\dot{\boldsymbol{u}}(t)$ and $\ddot{\boldsymbol{u}}(t)$ represent velocities and accelerations related to the structural motion, respectively. For the present case the principally feared seismic effect is related to the undulatory dynamic motion and, as a consequence, reference can be made to the following static condensation procedure. Let us consider as dynamically meaningful just the horizontal displacements of the structure nodes; in this case, with appropriate partition and reordering of the relevant operators, the free vibration equations of motion read:

$$\begin{pmatrix} \boldsymbol{M}_{tt} & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \ddot{\boldsymbol{u}}_t(t) \\ \ddot{\boldsymbol{u}}_r(t) \end{pmatrix} + \begin{pmatrix} \boldsymbol{K}_{tt} & \boldsymbol{K}_{tr} \\ \boldsymbol{K}_{rt} & \boldsymbol{K}_{rr} \end{pmatrix} \begin{pmatrix} \boldsymbol{u}_t(t) \\ \boldsymbol{u}_r(t) \end{pmatrix} = \begin{pmatrix} \boldsymbol{f}(t) \\ 0 \end{pmatrix}, \tag{4}$$