Engineering Structures 62–63 (2014) 65–74

Contents lists available at ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Vibration analysis of shear deformable shallow shells using the Boundary Element Method

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ARTICLE INFO

Article history:
Received 21 April 2013
Revised 24 December 2013
Accepted 6 January 2014
Available online 11 February 2014

Keywords:
Boundary Element Method
Shear deformable shallow shells
Dynamic harmonic analysis
Dual Reciprocity Boundary Element Method
Modal analysis
Harmonic excitation

ABSTRACT

In this work, the modal and harmonic analysis of elastic shallow shells, using a Dual Reciprocity Boundary Element formulation, is presented. A boundary element formulation based on a direct time-domain formulation using the elastostatic fundamental solutions was used. Effects of shear deformation and rotatory inertia were included in the formulation. Shallow shell was modeled coupling boundary element formulation of shear deformable plate and two-dimensional plane stress elasticity. Domain integrals related to inertial terms were treated using the Dual Reciprocity Boundary Element Method. Several examples are presented to demonstrate the efficiency and accuracy of the proposed formulation.

1. Introduction

Dynamic plate bending problems appear on civil, mechanical, aerospatial and naval applications. The complexity involved in the dynamic response of plates turns these problems in a challenging one from mathematical point of view. In general, numerical methods represent the only way to obtain approximate solutions for dynamic analysis. However, the use of traditional methods based on domain discretization requires refined meshes involving a high number of degrees of freedom, which requires a significant computational effort.

Nowadays, the Boundary Element Method (BEM) has emerged as an accurate and efficient numerical method for shear deformable plate and shell static analysis [1,8,18–20]. Dynamic analysis of shear deformable plates using elastodynamic fundamental solutions or Laplace or Fourier transformations of these fundamental solutions were used in [2,7,15–17]. In [5,13] a time-domain direct formulations based on elastostatic fundamental solutions for dynamic analysis of shear deformable plates, are presented. However, to the best of the author knowledge, the Boundary Element Method has not been used for the dynamic analysis of shear deformable elastic shallow shells.

This work presents free vibration and harmonic analysis of shear deformable elastic shallow shells using a boundary element formulation. This formulation is based on direct time integration and elastostatic fundamental solutions. Effects of shear deformation and rotatory inertia are included in the formulation. Shells were modeled by coupling the boundary element formulation for shear deformable plates based on the Reissner plate theory and two-dimensional plane stress elasticity, as presented in [1,19]. The Dual Reciprocity Boundary Element Method for the treatment of domain integrals involving inertial mass, was used. Numerical examples are presented and results were compared with those obtained using finite element models.

2. Shallow shell dynamic equations

Consider a shallow shell of uniform thickness h and mass density ρ, occupying the area Ω, in the x_1x_2 plane, bounded by the contour $\Gamma = \Gamma_w \bigcup \Gamma_q$ with $\Gamma = \Gamma_w \bigcap \Gamma_q \equiv 0$, as presented in Fig. 1. The dynamic bending response for the shallow shell was modeled coupling the classical Reissner plate theory and the two-dimensional plane stress elasticity as presented in [18].

Equations of motion for an infinitesimal plate element are given by [10]:

$$L^b_{ik} w_k + q^*_i = \Lambda^b_{ik} \ddot{w}_k + \Lambda^{bm}_{i\alpha} \ddot{u}_\alpha \quad (1)$$

$$L^m_{\alpha\beta} u_\beta = \Lambda^{bm}_{\alpha\beta} \ddot{w}_\beta + \Lambda^m_{\alpha\beta} \ddot{u}_\beta \quad (2)$$

Indicial notation is used throughout this work. Greek indices vary from 1 to 2 and Latin indices take values from 1 to 3. Einstein's summation convention is used unless otherwise indicated. In these

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http://dx.doi.org/10.1016/j.engstruct.2014.01.010
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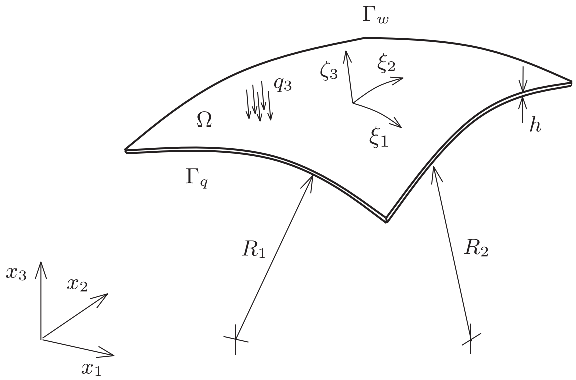

Fig. 1. Shallow shell geometry.

equations, w_α represents rotations with respect to x_1 and x_2 axes, and w_3 represents transverse deflection; $\ddot{w}_\alpha$ denotes angular accelerations with respect to x_1 and x_2 axes, respectively, $\ddot{w}_3$ represents the transverse acceleration; u_α and $\ddot{u}_\alpha$ represents membrane displacements and accelerations along x_α axis, respectively; Tensors Λ^b_{ij}, Λ^{bm}_{ij} and Λ^m_{ij} are defined as: $\Lambda^b_{\alpha\beta} = I_2\delta_{\alpha\beta}$ and $\Lambda^b_{33} = I_0$; $\Lambda^{bm}_{\alpha\beta} = I_1\delta_{\alpha\beta}$; $\Lambda^m_{\alpha\beta} = I_0\delta_{\alpha\beta}$; $\delta_{\alpha\beta}$ is the Kronecker's delta and I_i is the mass inertias given by:

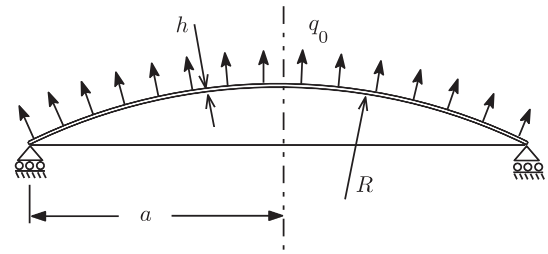

Fig. 3. Circular shallow shell geometry.

$$I_i = \rho \int_{-h/2}^{+h/2} \zeta^i d\zeta, \quad i = 0, 1, 2 \tag{3}$$

The differential operator L_{ik} in Eq. (1), is given by [1]:

$$L^b_{\alpha\beta} = \frac{D(1+\nu)}{2}\left[(\nabla^2 - \lambda^2)\delta_{\alpha\beta} + \frac{(1+\nu)}{(1-\nu)}\frac{\partial}{\partial x_\alpha}\frac{\partial}{\partial x_\beta}\right] \tag{4}$$

$$L^b_{\alpha 3} = -L^b_{3\alpha} = -\frac{(1-\nu)D}{2}\lambda^2\frac{\partial}{\partial x_\alpha} \tag{5}$$

$$L^b_{33} = -\frac{(1-\nu)D}{2}\lambda^2\nabla^2 \tag{6}$$

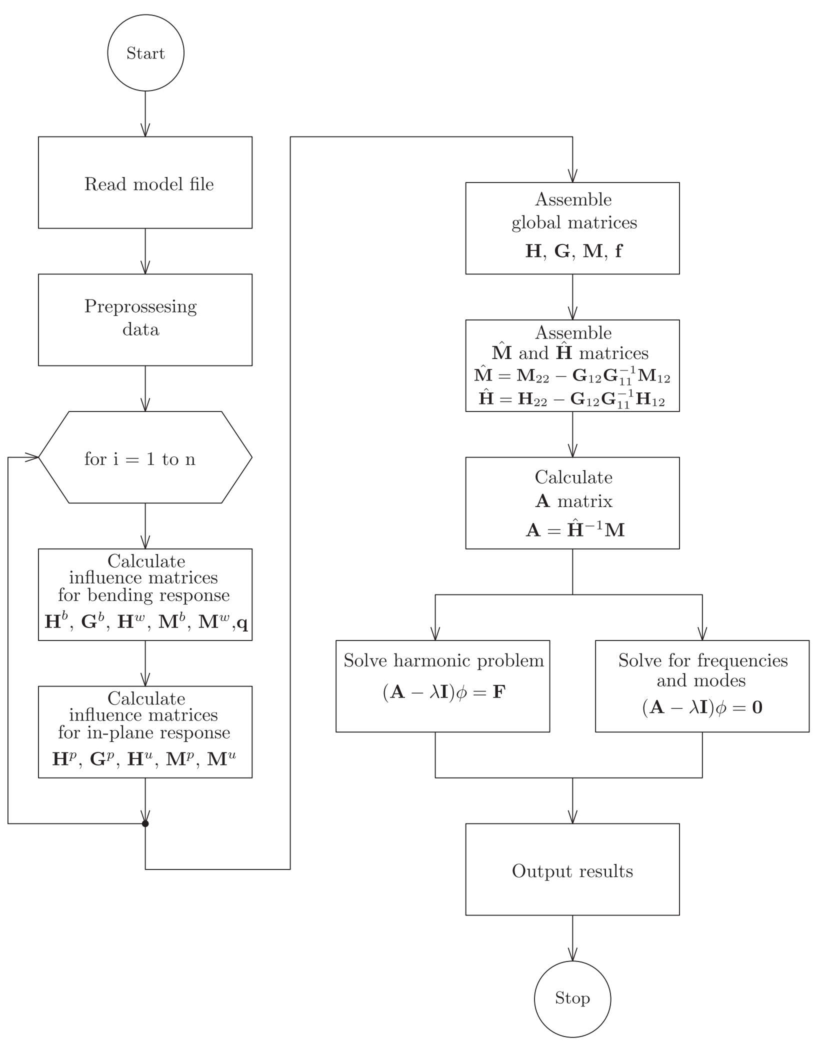

Fig. 2. Flow chart for computer program.