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Estimation of extreme value distribution of crosswind response of wind-excited flexible structures based on extrapolation of crossing rate

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ABSTRACT

This paper presents an approach based on response crossing rate analysis for estimating extreme value distribution of crosswind response of wind-excited structures with significant nonlinear aeroelastic effect. The crossing rates at various thresholds are calculated from response time histories, and are then curve-fitted by a prescribed parametric model. The influence of narrow band characteristic of response is accounted by using envelope process with two-state description of crossings and a further consideration of mean clump size. The curve-fitting and extrapolation of crossing rate permit estimation of extreme value distribution using Poisson distribution of crossings. The effectiveness and accuracy of the approach are examined using simulated crosswind responses covering a wide range of non-Gaussian characteristics, and also using full-scale vibration measurement data of a wind-excited traffic-signal-support structure. The results illustrated that the approach can produce robust estimations of extreme value distributions of hardening non-Gaussian crosswind responses. The narrow band characteristic of response process has very limited effect on the extremes of hardening non-Gaussian responses. The approach presented is especially effective in practice, where the number of available response time histories is often very limited, and a direct use of extreme samples fails to provide accurate estimation of extreme statistics.

1. Introduction

With the decrease in structural frequency, the crosswind response at the vicinity of vortex lock-in wind speed is of concern for structural design of super tall buildings and other dynamically sensitive structures. Attributed to nonlinear aeroelastic effect, the crosswind response at the vicinity of vortex lock-in speed exhibits characteristics of extreme value distribution distinctly different from those of traditional stochastic buffeting response. The peak factor, i.e., the ratio of mean extreme to standard variation (STD) of the response varies from $\sqrt{2}$ to around 4.5, which cannot be estimated from widely used Davenport formula (e.g., [29,1,22,2,31]). Chen [3] has revealed that the hardening non-Gaussian characteristic of response is responsible for the unique extreme value distribution and lower peak factor of crosswind response.

The translation process theory allows the estimation of extreme value distribution of a non-Gaussian process from that of a underlying Gaussian process for which an analytical formulation is available (e.g., [12,13]; [30]). The translation process theory has been widely used for softening non-Gaussian processes with kurtosis larger than 3, such as local dynamic wind pressures at flow separation regions, wind load effects considering the influence of squared turbulence, and vectorially combined structural accelerations (e.g., [18,14,10,4,19]). The applications of translation process theory for hardening non-Gaussian processes with kurtosis less than 3 can be found in Gioffre and Gusella [11] and Ding et al. [8]. Chen [3] has developed practical closed-form formulations for determining the Hermite translation function directly from the response kurtosis, which permit calculations of extreme value distribution and peak factor using response kurtosis and bandwidth parameter. The extensive study based on simulated crosswind response time history samples and full-scale vibration data have illustrated the accuracy of the proposed approach. While the translation process theory is generally very effective, the accuracy of prediction is affected by the probability distribution model or translation function model used in representing the distribution tail (e.g., [8]). For instance, the moment-based Hermite translation model may not well predict the extreme value distribution of a strong non-Gaussian process, while the translation function determined by mapping of the cumulative distribution function (CDFs) can provide a better estimation (e.g., [8]).

This paper presents an alternative approach for estimating extreme value distribution of hardening non-Gaussian crosswind response. The extreme theory of narrow band Gaussian process is revisited. It is followed by the discussion concerning the estimations of mean crossing rate of a non-Gaussian response process directly from its time history samples. The influence of narrow band

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feature on the crossing rate is taken into consideration by using the envelope process. The estimated crossing rates at different response thresholds are curve-fitted into a prescribed parametric model suggested in Naess and Gaidai [21]. The curve-fitting of the crossing rate allows an extrapolation of crossings into large thresholds and facilitates determination of extreme value distribution based on the Poisson distribution of crossings. While this theoretical framework has been used in other applications, its effectiveness and accuracy as applied to hardening non-Gaussian crosswind response of wind-excited structures have not yet been examined. In this study, application of this framework to crosswind response of wind-excited structures is first addressed. The response time histories are generated using an analytical model, which was developed based on wind tunnel and full-scale measurements and has been widely used in design codes and standards. The estimations of peak factor and coefficient of variation (COV) of extreme are compared with those directly estimated from response extreme samples. The confidence intervals of estimations are also determined to examine the performance of the approach. Finally, the approach is applied to the analysis of full-scale vibration measurements of a wind-excited traffic-signal-support structure.

2. Theoretical formulations

2.1. Upcrossing rate and extreme value distribution of a Gaussian process

The extreme value of a zero-mean stationary stochastic process $Y(t)$ during a time duration of T, i.e., $y_{\max} = \max\{y(t), 0 \leqslant t \leqslant T\}$, is considered. The CDF of $y_{\max}$ is given in terms of the mean upcrossing rate $\nu_y(y)$ as follows based on Poisson assumption of crossings:

$$F_{y\max}(y) = \exp[-\nu_y(y)T] \tag{1}$$

The mean upcrossing rate is related to probability distribution of the process by Rice formula ([23]):

$$\nu_y(y) = f_y(y)\int_0^{\infty} \dot{y} f_{\dot{y}|y}(\dot{y})d\dot{y} \tag{2}$$

where $f_y(y)$ is probability density function (PDF) of $Y(t)$; and $f_{\dot{y}|y}(\dot{y})$ is PDF of the derivative process $\dot{Y}(t)$ conditioned on $Y(t) = y$.

In the case of Gaussian process, $Y(t)$ and $\dot{Y}(t)$ are independent, and $\dot{Y}(t)$ is also a Gaussian process. Eq. (2) leads to

$$\nu_y(y) = \nu_0 \exp(-y^2/2\sigma_y^2) \tag{3}$$

where $\nu_0 = \nu_{y0} = \sigma_{\dot{y}}/2\pi\sigma_y = \sqrt{\lambda_2/\lambda_0}/2\pi$ is upcrossing rate of $Y(t)$ at the zero mean; σ_y and $\sigma_{\dot{y}}$ are STDs of $Y(t)$ and $\dot{Y}(t)$; λ_n is nth moment of the process power spectral density (PSD) function $S_y(f)$ defined as

$$\lambda_n = \int_0^{\infty} (2\pi f)^n S_y(f)df \tag{4}$$

The p-fractile value of extreme, $y_{p\max}$, where $F_{y\max}(y_{p\max}) = p$, is then calculated as

$$y_{p\max} = \sqrt{2\ln[\nu_0 T/\ln(1/p)]} \tag{5}$$

The mean and mean plus STD of extreme can be estimated as 57% and 86% fractile values, respectively, when the extreme value distribution is approximated as a Type I Gumbel distribution. Davenport [7] derived the following formulas for the mean and STD of extreme through closed-form expressions of extreme statistics:

$$\bar{y}_{\max} = \left(\beta + \frac{\gamma}{\beta}\right)\sigma_y \tag{6}$$

$$\sigma_{y\max} = \frac{\pi}{\sqrt{6}\beta}\sigma_y \tag{7}$$

where $\beta = \sqrt{2\ln(\nu_0 T)}$; and $\gamma = 0.5772$ is Euler's constant.

For very narrow band processes and also for relatively low crossing thresholds, the crossings tend to occur in cluster, thus the Poisson assumption of crossings is no longer adequate. Vanmarcke [28] introduced an improved crossing model to account for the crossing clustering using the envelope process with two-state descriptions of upcrossings and a further consideration of mean clump size. According to this model, the crossing rate $\nu_y(y)$ should be replaced with $\nu_{yE}^{(2)}(y)$, which is referred to as improved E-crossing rate and is given as

$$\nu_{yE}^{(2)}(y) = \nu_0 \exp\left(-y^2/2\sigma_y^2\right)\frac{\left[1-\exp\left(-\sqrt{2\pi}q_e y/\sigma_y\right)\right]}{\left[1-\exp\left(-y^2/2\sigma_y^2\right)\right]} \tag{8}$$

where $q_e = q^{1+b}$; and $b = 0.2$ is an empirically determined constant; and q is bandwidth parameter and is defined as

$$q = \sqrt{1-\lambda_1^2/(\lambda_0\lambda_2)} \tag{9}$$

By the way, the crossing rate of envelope process, $\nu_{yE}(y)$, referred to as original E-crossing rate, is given as ([27])

$$\nu_{yE}(y) = \nu_0 \exp\left(-y^2/2\sigma_y^2\right)\sqrt{2\pi}q_e y/\sigma_y \tag{10}$$

The p-fractile value of extreme, $y_{p\max}$, can be approximately calculated by setting $1-\exp\left(-y_{p\max}^2/2\sigma_y^2\right) \approx 1$:

$$y_{p\max} = \sqrt{2\ln\left\{y_p\left[1-\exp\left(-\sqrt{2\pi}q_e y_{p\max}\right)\right]\right\}} \tag{11}$$

It is further simplified by replacing $y_{p\max}$ on the right hand side of the equation with $y_{p\max} \approx y_e = \sqrt{2\ln y_p}$, where $y_p = \nu_0 T/\ln(1/p)$. It leads to the following semi-empirical equation ([27])

$$y_{p\max} = \sqrt{2\ln\left\{y_p\left[1-\exp\left(-\sqrt{2\pi}q_e y_e\right)\right]\right\}} \tag{12}$$

It is noted that when the mean and mean plus STD of extreme are estimated as 57% and 86% fractile values, use of the above semi-empirical formula with $q_e = q^{1.3}$ leads to estimations closer to those estimated using integration of extreme value distribution.

When the extreme value of $|Y(t)|$ is concerned, ν_0, y_p and $\sqrt{2\pi}y_e = \sqrt{4\pi\ln y_p}$ in the proceeding formulations related to the single-barrier crossing, i.e., B-crossing, should be replaced by $2\nu_0$, $2y_p$ and $\sqrt{\pi/2}y_e = \sqrt{\pi\ln(2y_p)}$, respectively, for the double-barrier crossing, i.e., D-crossing.

2.2. Crossing rate and extreme value distribution of a non-Gaussian process

The mean upcrossing rate and extreme value distribution of a stochastic non-Gaussian response process generally cannot be calculated using closed-form formulations based on Rice's formula, because the joint probability distribution of the process and its derivative is generally unknown. In the case of translation non-Gaussian process, a monotonic translation function model can be established to relate the non-Gaussian process with a underlying Gaussian process (e.g., [12,13,30]). The Hermite polynomial function model is often used for the translation function, which can be determined by first four statistical moments or mapping of CDFs. The crossing rate is then calculated from that of the Gaussian process and the extreme value distribution is subsequently determined. In the case of a hardening non-Gaussian response process concerned in this study, the underlying Gaussian process is expressed as a Hermite function of non-Gaussian process as

$$u = g^{-1}(y_0) = h_1\left[y_0 + h_3(y_0^2-1) + h_4(y_0^3-3y_0)\right] \tag{13}$$

where $y_0 = y/\sigma_y$.