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Integrity Index and Integrity-based Optimal Design of structural systems

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ABSTRACT

The reliability of a system is a function of the reliability of each of its components. This paper develops the concept of integrity of a system as a measure of the balance between the reliability of the system components and puts forward the definition of Integrity Index as well as a method to maximize this index in a structural system. The integrity is quantified using the conditional probabilities of failure of the components given that the system has failed to assess the importance of the individual components in system reliability. The Integrity Index is defined as 1 minus the difference between the maximum and minimum conditional probabilities of failure of the system components. Therefore, this index can quantify the integrity of any structural system as long as the conditional probabilities of failure of its components can be calculated. Integrity Index is equal to 1 when all of the system components have the same contribution to the system reliability and it is 0 when there is the maximum imbalance between the contributions of the components to the system reliability. Using the conditional probabilities of failure of the components, a new method is then developed for Integrity-based Optimal Design that maximizes the system integrity giving the maximum Reliability Return on Investment (RROI). This method is first discussed for various systems (series, parallel and general). Then, as an example, the Integrity Index and Integrity-based Optimal Design are presented for an offshore mooring system.

1. Introduction

The design of most engineering structures involves an optimization process to minimize the project costs while achieving the reliability requirements. Such optimization process becomes more important in more expensive and complex systems where the long term reliability of the structure justifies the investment. In practice, the reliability of most engineering structures is achieved by using partial safety factors that locally (i.e., for each component) achieve the safety. However, such approach can lead to an unbalanced design where some components are under- and some over-designed. Because this design method does not compare the reliability of the components and does not quantify their contributions to the reliability of the system, it can produce a system that is missing what we call the design integrity, which is a measure of the balance between the contributions of the system components to the reliability of the system.

Different measures have been proposed in the literature to assess the robustness or optimality of structural systems. However, these measures do not quantify directly the integrity. For example, the redundancy of structural systems have been assessed using the relative reliability index and system reserve ratio in bridge structures [1–3] and redundancy and damage factors have been proposed for trusses [4,5]. However, such measures mainly quantify the level of redundancy in the structure by comparing the system reliability index with that of the components using probabilistic analysis together with assessing the relative impact of failure of a component on the system resistance based on deterministic analyses. Some other studies have proposed indices that indirectly assess the robustness of the system through reliability analysis [6]. Overall, the limitation of these measures is that they do not provide a direct evaluation of the safety of components considering their importance to the system safety, where the system safety can be in terms of survival or serviceability. Therefore, an approach for quantifying the integrity of a system is needed.

A structure has a low integrity when the failure of some of its components is more likely to be the cause of the system failure than the failure of other components. This usually indicates that the structure could maintain the same level of safety at a lower cost by replacing the components that are designed too conservatively with less conservative but still safe alternatives. An integrity assessment based on this definition can also identify the weak components of the structure that are such that improving them can considerably increase the system reliability giving the highest reliability return on investment (RROI). Such integrity assessment

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requires a comparative study of the reliability of the system components considering their influence on the system reliability.

We quantify the integrity of a structure by assessing the importance of all the individual components in the system. Accurate quantification of this importance needs a probabilistic study because the component reliabilities depend on random variables (e.g., loads, material strengths.) Specifically, we use the conditional probability of failure of a component given system failure to quantify the importance of that component to the system reliability. This conditional probability is a measure of the likelihood that the system fails because of the failure of that component. We define the Integrity Index as 1 minus the difference between the maximum and minimum conditional probabilities of failure of the system components. Therefore, the Integrity Index exposes the variability of the reliability of the system components in terms of their importance concerning system failure. This index can be used to quantify the integrity of any structure as long as the conditional probabilities of failure of its components can be calculated.

Because the conditional probabilities of failure of the system components are by definition between 0 and 1, the Integrity Index also takes values between 0 and 1. For an Optimal Design, the Integrity Index is 1, which means that all of the system components have the same contribution to the reliability of the system. Conversely, if the Integrity Index of a system with several components is close to 0, there is a significant imbalance between the contributions of the components to the system reliability. This imbalance can be fixed if the system components are redesigned or their configurations and arrangements in the system are altered. Calculating the Integrity Index provides a basis to evaluate the effect of any changes on the design integrity.

Improving the integrity of a design can be conducted through an Integrity-based optimization, here named Integrity-based Optimal Design. For example, in the case of a series system in which the conditional probability of failure of a component is considerably higher than the conditional probabilities of failure of the other system components, the system's overall reliability can be improved by either improving the reliability of this component or changing the topology of the system to make it, for example, a parallel system at least to some degree, providing some level of redundancy. Using partial safety factors at the component levels does not capture accurately the importance of a component to the system reliability. As a result, each component is designed individually and there are no considerations to the role of that component in the system. On the other hand, when a component has a considerably lower conditional probability of failure than other system components, which means it is safer than other components, its safety can be reduced (e.g., it can be replaced with a weaker/less expensive option) without necessarily affecting the reliability of the system, because its additional safety does not increase the safety of the system. The ultimate goal of an Integrity-based optimization is to reduce the differences between the conditional probabilities of failure of the system components maximizing the Integrity Index. The proposed method of quantitative integrity analysis does not account for the difference of the costs of the components and therefore cannot be used directly to minimize the costs of a design; however, it is especially applicable where the costs and consequences of a system failure are considerably greater than the cost of its individual components, such as some offshore systems; the failure of which could result in major damage to the environment or profit loss due to a production delay. For such systems, quantitative integrity analysis can identify the components with higher risks or significance for system safety so that proper decisions can be made, either in their design or their inspection and maintenance planning. When needed, incorporating cost considerations in the analysis is also possible and straightforward.

An important field of application of the proposed Integrity Index and Integrity-based Optimal Design is the design of offshore mooring systems. Offshore mooring systems consist of several components including multiple line segments, clumps or buoys, and the foundations (anchors, suction caissons, etc.) that form several series subsystems. The environmental loads and the structural capacities of a mooring system are random variables. As a result, probabilistic methods are ideally suited to design offshore mooring systems. As an illustration, the proposed Integrity Index and Integrity-based Optimal Design are used in this paper for the design of a mooring system.

2. Integrity Index

The Integrity Index equals 1 minus the maximum differences of the conditional probabilities of failure of the system components given system failure. We propose the following equation to calculate the Integrity Index, I:

$$I = 1 - \left\{ \underset{i=1,\dots,N}{\mathrm{Max}} [\mathrm{P}(g_i \le 0 | g_s \le 0)] - \underset{i=1,\dots,N}{\mathrm{Min}} [\mathrm{P}(g_i \le 0 | g_s \le 0)] \right\} \quad (1)$$

where g_s is the system limit-state function, g_i is the limit-state function of component i, N is the number of components in the system, $\mathrm{P}(g_i \leqslant 0|g_s \leqslant 0)$ denotes the conditional probability of failure of component $i(g_i \leqslant 0)$ given system failure $(g_s \leqslant 0)$, and Max and Min refer to maximum and minimum respectively. Eq. (1) indicates that a system has the highest integrity (i.e., $I = 1$) if all its components have equal conditional probabilities of failure given system failure and minimum integrity (i.e., $I = 0$) in case of maximum disparity in the values of the conditional probabilities.

2.1. Calculating the conditional probability of failure in a structural system

The first step in calculating the Integrity Index of a structural system is to calculate the conditional failure probabilities for all its components given that the system failed. The conditional probability of failure of a component given the system failure can be calculated using the Bayes' Rule [7] as

$$\mathrm{P}(g_i \le 0 | g_s \le 0) = \frac{\mathrm{P}(g_s \le 0 | g_i \le 0)\mathrm{P}(g_i \le 0)}{\mathrm{P}(g_s \le 0)} \quad (2)$$

Eq. (2) indicates that the conditional probability of failure of component i is a function of the probability of failure of the system, the probability of failure of component i, and the conditional probability of failure of the system given component i failed. In practice, the failures of a structural system and its components depend on the demands (environmental loads) and the resulting internal forces as well as the capacities (strengths) of the components. An approach to calculate these probabilities is by considering fragility curves [8] defined as

$$\mathrm{P}(g_i \le 0 | d_E) = \mathrm{P}(c_i \le d_i | d_E) \quad (3)$$

where c_i is the capacity (e.g., strength or allowable deformation) of component i and d_i is the demand on component i associated to the system demand d_E due to the environmental loads. If the Probability Density Function (PDF) of the capacity of component i is available and d_i is calculated using structural analysis methods (e.g. FEM) based on d_E, Eq. (3) can be used to find the conditional probability of failure of component i given the demand d_E.

The conditional probabilities of system failure given d_E, $\mathrm{P}(g_s \leqslant 0|d_E)$, and the conditional probabilities of system failure given d_E and failure of component i, $\mathrm{P}(g_s \leqslant 0|g_i \leqslant 0, d_E)$, can be calculated similarly. Then using Eq.(2) and the total probability rule