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An improved equivalent linear model of seismic isolation system with bilinear behavior

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ABSTRACT

Equivalent linearization of seismic isolation systems is often recommended in modern structural specifications for the purpose of simplification. In this paper, limited conditions specified in equivalent linearization of seismic isolation system are investigated when subjected to seismic loads. A large number of numerical simulations, including both approximate linear and exact nonlinear analyses, are performed using a program specially developed by MATLAB in conjunction with OpenSees. Approximate to exact maximum displacement ratio averaged over seven earthquake ground motions is selected as the evaluation indicator. Results reveal that, although satisfactory estimates can be obtained if the limited conditions are met, the application scope of equivalent linearization of seismic isolation system is significantly restricted. To improve the prediction accuracy of equivalent linear analysis in a wider parameter space, equivalent linear model recommended in codes is modified through introducing a factor, which is related to ductility ratio, post-to-pre yield stiffness ratio and initial period of seismic isolation system. It is demonstrated that the newly proposed equivalent linear model yields more accurate results when compared with other equivalent linear models.

1. Introduction

With rapid development of seismic isolation system, nowadays, it has been one of the most popular methods to mitigate seismic risk [1–4] and can be easily applied to new buildings or used for the retrofitting of existing buildings. Isolation system provides a relatively flexible layer such that the fundamental period of seismically isolated building is elongated. In addition, with the nonlinear hysteretic behavior, seismic isolators dissipate most of input energy when subjected to seismic ground motions. Due to these advantages, the superstructure generally remains linear elastic and moves like a rigid body. Therefore, seismically isolated buildings can be simulated by a single-degree-of-freedom (SDOF) system, especially for low rise buildings.

Isolation bearings are expected to exhibit high initial elastic stiffness but low post-yield stiffness. For small loading caused by - wind or low-intensity earthquake, the building is designed to yield small deformations and keep intact. However, when the force caused by relatively large earthquakes in seismic bearing is beyond its yield strength, hysteretic deformations produced due to the low post-yield stiffness will be used to withstand the seismic ground motions. As a result, seismic isolation systems often exhibit strong nonlinearity. Different nonlinear mechanisms may exist in various seismic isolation devices. Many sophisticated hysteretic models [5–11] have been developed to simulate these nonlinear behaviors. However, to simplify the design procedure, bilinear force–deformation behavior is generally recommended in modern structural specifications, which can be characterized by the initial elastic stiffness K_i, the post-to-pre yield stiffness ratio α, and the yield strength F_y, as presented in Fig. 1.

Regarding analysis methods of seismically isolated buildings, both linear and nonlinear methods are proposed in most of modern specifications. Of course, due to the remarkably increased computing resources, nonlinear time history analysis (NLTH) can be easily performed. But, solving of a system with a large number of degrees of freedom may require an exorbitant amount of time when time history analysis methods are used. Also, the enormous amount of output results from such a system may be so detailed that it is impractical for engineers to summarize [12]. In the initial stage of structural design, many structural configurations are not well-defined such as arrangement and properties of seismic isolators. So, approximate methods of analysis of seismic isolation systems are generally applied. With several limited conditions, isolation system may be modeled with equivalent linear (EL) visco-elastic behavior, which is often used to approximate nonlinear behavior of isolation system and to avoid a large amount of computational time and

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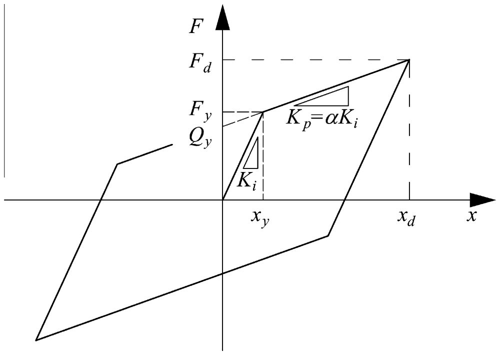

Fig. 1. Idealized bilinear hysteresis model of seismic isolators.

effort. In this manner, the original nonlinear system can be replaced by the EL system in the process of structural design.

According to different treatments in estimating equivalent period and equivalent viscous damping ratio, several analytical or empirical formulas used in EL analysis were developed in the past decades [13–17]. The method proposed by Rosenblueth and Herrera [13] was most commonly used and had been adopt by Eurocode 8 [18], AASHTO [19] and NTC 2008 [20]. In order to assure the suitability of equivalent linearization of seismic isolation system, several limited conditions are specified in these specifications. For the sake of brevity, the model proposed by Rosenblueth and Herrera is abbreviated to R&H model hereafter. Detailed introduction and evaluation of different EL methods could be found in recent study of the authors [21], where fourteen EL methods proposed in the literatures were assessed based on single-degree-of-freedom (SDOF) systems with bilinear hysteretic behavior. However, in that study the post-to-pre yield stiffness ratio was fixed to 0.1, thereby the influence of post-to-pre yield stiffness ratio on the accuracy of EL analysis method had not been identified. Furthermore, as mentioned by Liu et al., optimal EL properties may exist between the methods proposed by Dicleli and Buddaram [15] and Guyader and Iwan [17], but they did not give any specific formula to improve the prediction accuracy.

The main objectives of this study are: (i) to present the limited conditions of equivalent linearization specified in structural specifications; (ii) to perform a comprehensive evaluation of the limited conditions through comparison of results between EL and NLTH analysis and (iii) to improve the estimation accuracy and to expand the application scopes of equivalent linearization technique by modifying R&H model. Detailed limitations in performing equivalent linearization of seismic isolation system are described in Section 2. Section 3 presents the selected earthquake ground motions. Then, in Section 4, parameter variation and assessment procedure are described as well as a specially developed program. Results of parametric analyses are examined in Section 5. Section 6 proposes a new EL model through introducing a factor to R&H model. Conclusions are given in Section 7.

2. Equivalent linearization of seismic isolation system

For a SDOF system with bilinear hysteretic behavior as presented in Fig. 1, the equation of motion under earthquake excitation is given by:

$$M\ddot{x}(t)+C\dot{x}(t)+F(x(t),\dot{x}(t))=-M\ddot{x}_g(t) \tag{1}$$

where $\ddot{x}(t),\dot{x}(t)$, and $x(t)$ are the acceleration, velocity, and displacement, respectively, of the SDOF system relative to the ground; C is the damping coefficient; $F(x(t),\dot{x}(t))$ is the restoring force; M is the mass of the system; and $\ddot{x}_g(t)$ is the ground acceleration.

Dividing by the mass, Eq. (1) may be rewritten as:

$$\ddot{x}(t)+\frac{4\pi\xi_0}{T_0}\dot{x}(t)+\frac{F(x(t),\dot{x}(t))}{M}=-\ddot{x}_g(t) \tag{2}$$

where ξ_0 is the inherent viscous damping ratio and T_0 is the initial natural period, which is given by:

$$T_0=2\pi\sqrt{M/K_i} \tag{3}$$

The underlying premise of equivalent linearization is that the inelastic response of an isolation system can be adequately modeled using a fictitious viscously damped elastic system whose stiffness and damping characteristics are selected such that the maximum displacement responses of the two systems are approximately equal, as shown in Fig. 2.

Therefore, replacing the bilinear system with a linear viscous damped system, the differential equation of motion may be expressed as:

$$M\ddot{x}_{eq}(t)+C_{eq}\dot{x}_{eq}(t)+K_{eq}x_{eq}(t)=-M\ddot{x}_g(t) \tag{4}$$

where $\ddot{x}_{eq}(t),\dot{x}_{eq}(t)$, and $x_{eq}(t)$ are the acceleration, velocity, and displacement, respectively, of the EL system relative to the ground; C_{eq} is the equivalent damping coefficient; K_{eq} is the equivalent stiffness; M is the mass of the system; and $\ddot{x}_g(t)$ is the ground acceleration.

Similarly, dividing by the mass, Eq. (4) may be rewritten as:

$$\ddot{x}_{eq}(t)+\frac{4\pi\xi_{eq}}{T_{eq}}\dot{x}_{eq}(t)+\left(\frac{2\pi}{T_{eq}}\right)^2x_{eq}(t)=-\ddot{x}_g(t) \tag{5}$$

where ξ_{eq} is the equivalent viscous damping ratio,

$$\xi_{eq}=\frac{C_{eq}}{2\sqrt{K_{eq}M}} \tag{6}$$

and T_{eq} is the period of vibration of the equivalent system,

$$T_{eq}=2\pi\sqrt{M/K_{eq}} \tag{7}$$

When the EL analysis is performed, it is obviously noted that appropriate estimation of EL properties is crucial for the accuracy of results. In R&H method, equivalent stiffness K_{eq} can be determined based on the concept of secant stiffness, as presented in Fig. 3(a),

$$K_{eq}=\frac{1+\alpha(\mu-1)}{\mu} \tag{8}$$

where μ is the displacement ductility ratio and defined as:

$$\mu=x_d/x_y \tag{9}$$

Employing the equal energy dissipation principle, first proposed by Jacobsen [22], the hysteretic damping ratio of the linear elastic system can be derived, as shown in Fig. 3(b),

$$\xi_{hyst}=\frac{E_H}{4\pi E_S}=\frac{4Q_y(x_d-x_y)}{2\pi K_{eq}x_d^2} \tag{10}$$

where E_H is hysteretic energy dissipated per cycle of motion through inelastic deformation, E_S is the elastic strain energy.

Considering the post-to-pre yield stiffness ratio and the displacement ductility ratio, equivalent viscous damping ratio can be expressed as:

$$\xi_{eq}=\xi_0+\xi_{hyst}=\xi_0+\frac{2(1-\alpha)(\mu-1)}{\pi\mu[1+\alpha(\mu-1)]} \tag{11}$$

However, specified in many specifications, nonlinear behavior of seismic isolation system may be considered as being equivalent linear only if all the following requirements are satisfied: