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# Limits to the strength design of reinforced concrete shells and slabs

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ABSTRACT

One of the most important works in the ultimate limit state design of reinforced concrete plates or shells subjected to flexure and membrane actions is the one provided by Brondum-Nielsen (1974). Therein, the author divides the shell element into three layers; the outer layers withstand a state of membrane forces located on their middle surfaces. The forces at the centroid of the reinforcement, in both directions, have been obtained from equilibrium, and the steel area needed is computed by dividing these tension forces by the steel yield stress, $f_y$. An extension to the strain plane hypothesis widely used in the strength design of RC beams and columns is presented, aiming at RC strength design of shells and slabs. As a result, limits to the application of the Bromdum-Nielsen procedure are given in this work since it cannot always be guaranteed that the stress in the steel is $f_y$ as the original method proposes. A new method based on the computation of the balance point in the beam flexure design is developed to check the limits of application of Brondum-Nielsen's approach. The Upper Bound Theorem of plasticity guaranties that the obtained forces are on the safe side. Examples are provided.

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## 1. Introduction

Capacity of reinforced concrete (RC) shells and slabs has always been an interesting topic, e.g. [1–3]. The problem of designing the reinforcement for a concrete plate or shell in ultimate limit state to withstand membrane forces together with bending and torsion moments has not yet been universally solved. As a consequence, the main RC design codes – Eurocode 2 [4], AC1 318 [5] – do not provide a general method to deal with this problem as they do with the beam cross section design. Only the Model Code CEB-FIP 2010 (MC2010) [6] states, literally, that "shell elements may be modelled as comprising three layers. The outer layers provide resistance to the in-plane effects of both the bending and the in-plane axial loading, while the inner layer provides a shear transfer between the outer layers." But the designer would have difficulty finding further information on this issue.

You can find many different techniques in literature that try to obtain a generally accepted solution. One of the first practical approaches to this problem is the report by Brondum-Nielsen [7]. This work deals with the shell element as if it were a sandwich element composed of three layers, with the outer layers being responsible for withstanding the membrane force decomposition of the external bending, torsion, in-plane axial and in-plane shear loading. Each of these layers contains an orthogonal reinforcing net. Gupta [8] takes the work of Brondum-Nielsen as a reference to propose a general solution based on an iterative trial-and-error design method using the principle of minimum resistance by also dividing the shell into three layers containing the orthogonally provided reinforcement. Marti [9] assigns the out of plane shear to the middle layer, complementing the work of Brondum-Nielsen. Lourenço and Figueiras [10,11] formulated the problem of reinforcing elements subjected to membrane and flexural forces based on equilibrium conditions and suggested a new iterative procedure. They have developed a consistent solution to the problem analyzing the shell element as a whole and not as two membrane outer layers. The information concerning this approach has been compiled by Fall et al. [12] in their revision of procedures of reinforcing methods in RC tailor-made structures. A similar approach to the one presented by Gupta is implemented in an iterative numerical computational algorithm by Min [13] and tested in several experimental examples. Furthermore, nonlinear inelastic analyses are performed using the Mahmoud-Gupta's computer program [14–17] to prove the adequacy of the presented equations. A similar formulation of the problem is adopted by Tomás and Martí [18] in order to mathematically optimize the amount of reinforcement in each finite element of the mesh that models the geometry of the problem, employing the summation of the tensile forces in the reinforcement as the objective function. One of the most recent works in this field is the one proposed by Bertagnoli et al. [19] where the authors provide a method based on sandwich layers to

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**Nomenclature**

| | |
|---|---|
| $a$ | distance between the middle surfaces of the top and bottom layers |
| $a_b$, $a_t$ | distances between the middle surfaces of bottom and top layers to the middle surface of the shell, respectively |
| $c_b$, $c_t$ | depth of bottom and top layers, respectively |
| $c_{b0}$ | first approximation of $c_b$ |
| $c_{k\mathrm{ilim}}$ | limit depth of layer $k$ ($k = t$ for top layer; $k = b$ for bottom layer) in order to yield the reinforcement in $i$ direction ($x$ or $y$) placed in the opposite layer $j$ |
| $d$ | depth of the reinforcement |
| $f_c$ | concrete compression strength |
| $f_y$ | yield stress of the reinforcement |
| $E_s$ | Young's modulus of the steel |
| $e_k$ | distance between the middle layer of the shell element to the centroid of the reinforcement placed in layer $k$ |
| $h$ | depth of the shell element |
| $z_{ya}$ | lever arm of $N_{yat} + N_{yab}$ related to the center of gravity of the gross section |
| $z_{yat}$ | lever arm of $N_{yat}$ related to the center of gravity of the gross section |
| $z_{yab}$ | lever arm of $N_{yab}$ related to the center of gravity of the gross section |
| $z_{xa}$, $z_{xat}$, $z_{xab}$ | idem for $N_{xat} + N_{xab}$, $N_{xat}$ and $N_{xab}$ |
| $M_a$ | flexural moment considered for the first estimation of $c_b$ |
| $M_x$, $M_y$ | bending moments in $x$ and $y$ directions applied to the shell element |
| $M_{xy}$ | twisting moment applied to the shell element |
| $N$ | axial force |
| $N_x$, $N_y$ | normal forces in $x$ and $y$ directions applied to the shell element |
| $N_{xy}$ | shear force applied to the shell element |
| $N_{xk}$, $N_{yk}$ | membrane normal forces in $x$ and $y$ directions in layer $k$ |
| $N_{xyk}$ | membrane shear force in layer $k$ |
| $N_{xak}$, $N_{yak}$ | tension forces in reinforcement placed in $x$ and $y$ directions in layer $k$ |
| $N_{ck}$ | concrete compression force in layer $k$ |
| $N_{total,k}$ | $N_{total,k} = N_{xak} + N_{yak}$. Summation of tension forces in the reinforcement placed in $x$ and $y$ directions in layer $k$ |
| $\alpha_k$ | angle between crack and $x$ direction, in layer $k$ |
| $\varepsilon_{cu}$ | concrete ultimate compressive strain |
| $\varepsilon_{j\text{-}i}$ | steel strain in $i$ direction, placed in layer $j$, when the depth of compression block in layer $k$ is $c_k$ |
| $\varepsilon_{j\text{-ilim-}\alpha k}$ | strain measured, in the direction of the crack of layer $k$ ($\alpha_k$), at the level of centroid of reinforcement placed in layer $j$ corresponding to the yield of the steel in $i$ direction |
| $\varepsilon_y$ | tension yield strain of the reinforcement |
| $\lambda$ | stress block factor of the rectangular stress distribution in concrete according Eurocode 2 |
| $\sigma$ | stress |
| $\alpha_k$ | angle between the crack in layer $k$ and $x$-direction |
| $\chi_{klim}$ | depth of the balance point |

*Subscript*

| | |
|---|---|
| $a$ | steel |
| $b$ | bottom layer |
| $i$ | $x$ or $y$ direction |
| $j$ | layer $j$, opposite layer to layer $k$ |
| $k$ | layer $k$ (top or bottom layer) |
| $x$ | direction $x$ |
| lim | balance conditions |
| $y$ | direction $y$ |

*Superscript*

| | |
|---|---|
| $*$ | actual value |

optimize the amount of reinforcing steel to be placed in the two outer layers. The method considers non-orthogonal reinforcement layouts, and the optimization procedure is based on genetic algorithms.

It is extraordinary that, despite all the aforementioned works, one of the most powerful and popular commercial pieces of software in structural design, the SAP2000©, uses the very first of one of these methods (Brondum-Nielsen's approach) to design the reinforcement of concrete shells in ultimate limit state under bending and in-plane axial forces [20].

Apart from dividing the shell element into some layers, all the presented works have another aspect in common with respect to the stresses in the reinforcement and in the concrete. The compressive stress in concrete – compression struts – should be distributed uniformly throughout the depth of the layer and the steel in tension is assumed to be yielded, – i.e. with stress equal to $f_y$; the later hypothesis is also known as limit-analysis solution. Both the tensile stress in concrete and the compressive stress in the reinforcement are neglected.

The part concerning the yielding of the reinforcement is found to be questionable by the authors of this study. Similarly, as in the case of ultimate state of bending in beams, where the plane sections hypothesis has to be satisfied, the strain in the reinforcement of one of the outer layers in a slab element should be related to the depth of the compression stress block in the opposite one.

This paper presents a necessary hypothesis to the strength design of reinforced concrete shells and slabs. Furthermore Marti [9] expresses an attempt to limit the applicability of Brondum-Nielsen's method "… these equations are only valid if the concrete compressive strength of the sandwich cover is not exceeded" – he calls cover to the thickness of the layer. His attempt, although interesting, is an inaccurate observation that limits his design to small axial forces and he fails to provide a basic understanding of the behavior of steel. Nonetheless an extension of the well established assumptions considered in the strength design of beams under bending is indeed a good advance in the reinforced concrete design of slabs and shells.

The present work draws on the formulation of the problem given in Brondum-Nielsen's procedure [7] to set the domains where this approach is valid. Firstly, Brondum-Nielsen's method is explained in a more compact fashion and the paper is therefore self-contained. Later, the beam balance point analogy is stated in order to determine reasonable limits to the application of Brondum-Nielsen's method. Finally, the original example given in Brondum-Nielsen's report is explained and the limits of application are checked.

## 2. Membrane forces decomposition of externally applied loads

The concrete shell element considered in this work has to withstand the established normal forces $N_x$ and $N_y$, the shear force $N_{xy}$, the bending moments $M_x$ and $M_y$ and the twisting moment $M_{xy}$. These actions are given per unit of length. Actions are considered positive if they are directed as indicated in Fig. 1(a). The shell element has one or two parallel layers of orthogonal reinforcing net of which the position is known. The depth of the shell element is $h$.

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