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A shear locking-free spatial beam element with general thin-walled closed cross-section

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ABSTRACT

A new shear locking-free spatial beam element with general closed thin-walled cross-section is presented in this paper. Based on the Timoshenko beam theory and Benscoter thin-walled beam theory, the proposed element considers the effects of shear deformation, coupled flexure and torsion, and warping shear stress. The shear locking is avoided by introducing an interior node to the element and adopting two-node Hermitian interpolation functions for transverse displacements and torsional rotation and three-node Lagrangian interpolation functions for the flexural rotations and warping function. Several examples are analyzed to validate the accuracy and convergent efficiency of the proposed beam element, and the results are compared with theoretical and numerical solutions. The comparisons demonstrate that the proposed element is applicable to the analysis of a thin-walled beam with an arbitrary closed cross-section.

1. Introduction

Development of thin-walled beam elements has aroused intensive interest from many researchers in the past few decades. Finite element modeling of the flexural problems was usually based on the theory of Bernoulli–Euler beam, and much attention has been paid to the influence of shear deformation on the behavior of thin-walled beams [1] involving problems with bending [2], restrained torsion [3–6] and buckling [7,8].

On the basis of the assumption that transverse shear strain keeps constant along the height of cross-section, the Timoshenko beam theory [9] includes the influence of shear deformation. Its application in the finite element area, termed as the Timoshenko beam element, usually features C0 type continuity [10–12]. When transverse displacements and flexural rotations are linearly interpolated, the Timoshenko beam element will behave very stiff with the large length-to-height ratio, which is known as the shear locking [13]. The locking results from additional spurious constraints [14], produced by the inconsistency of the interpolation used for transverse displacements and flexural rotations [14]. To avoid the locking, several approaches have been proposed in the literature, including the selective/reduced integration [14–18], assumed strain method [19–22], high order shear theory [13,23], Modified Hermitian shape functions [24–27], Consistent interpolation [13,28], finite element formulation based on curvature [29,30], hybrid/mixed formulation [31–35] and mesh-free/methless method [36–39]. In fact, the curvature-based element can be regarded as a model in which stress is interpolated, thus belonging to the category of hybrid/mixed elements [30]. In this paper, only the displacement-based finite element models are mainly concerned and the hybrid/mixed models or meshfree models will not receive any attention. For the selective/reduced integration, it means that the shear stiffness is integrated with fewer integration points than necessary to improve the performance of an element [14]. Its severe drawback is that an element adopting this approach suffers from instabilities generated by the hourglass modes [18]. For assumed strain method, strains are actually imposed under the discrete form and the relationship between displacements and shear strains is modified [18]. Actually, the B-bar methods proposed by Hughes [40] and Simo and Hughes [41] and the Discrete Shear Gap (DSG) approach proposed by Bletzinger et al. [42] and Koschnick et al. [43] can also be classified as the category, in which it is difficult and problematic that feasible sampling points are appropriately chosen [18]. Reddy [13] and Murthy et al. [23] proposed the high order shear theory, in which the transverse shear strain is represented as a quadratic function along the height of a beam's cross-section to alleviate shear locking. As far as thin-walled beams are concerned, they usually have complex forms of cross-section and approaches

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based on the high order shear theory seem to be too complicated to be applied. For the approach of modified Hermitian shape functions called Interdependent Interpolation Element (IIE) by Reddy [13], two constants are introduced in the Hermitian interpolation polynomials for the transverse displacements to include the shear deformation. These constants will vanish when the beam length is infinitely long and shear locking can be avoided. But when warping shear stress is included, this approach is inapplicable to thin-walled beams with closed cross-section, as will be discussed specially in the fourth paragraph in this section. For consistent interpolation, polynomials of degree m for transverse displacements and polynomials of degree m-1 for flexural rotations are used and the degrees of freedom (DOFs) of the middle node are generally eliminated [13]. To date, the approach is adopted only to avoid transverse shear locking and has not been found in the literature to be applied to thin-walled beams to include the influence of warping shear stress (to be discussed further in the fourth paragraph).

As is well known, a typical characteristic of a thin-walled beam is that the flexural and torsional deformations would be coupled when transverse load is applied along the asymmetric principal axis of its cross-section [1]. Gunnlaugsson and Pedersen [44] have contributed a lot to the development of the coupled flexure and torsion in the finite element formulation [45]. Hu et al. [45] and Kim and Kim [1], according to the kinematical description of a general thin-walled cross-section under load, formulated the transverse displacement including the flexural and torsional coupling. Many literatures [46–49] in the last decade can be found on the coupled torsional–flexural vibration of thin-walled beams.

Another common concern with a thin-walled beam is the non-uniform torsion (restrained torsion), and the associated linear elastic theory was developed by Vlasov [50] half a century ago. Warping shear stress is produced in the non-uniform torsion and it influences the torsional behavior of a thin-walled beam especially when the cross-section is closed. Benscoter [51] improved the Vlasov's theory and proposed a new theoretical frame for closed thin-walled beams, on the basis of which some methods were developed by a few researchers to include the effect of warping shear stress in the finite element formulation. These approaches can be categorized into two groups as (1) Homogeneous solutions of governing differential equations being adopted as the element interpolation functions [45], which results in a very complicated form of the element stiffness matrix. (2) Modified Hermitian shape function [27], in which a constant is added to the Hermitian interpolation polynomial for the torsional rotation. However, the constant is derived from a thin-walled beam with an open cross-section and this approach is not applicable to a closed thin-walled beam. For closed thin-walled beams, it is necessary that a new approach be presented to include the effect of warping shear stress. In this paper, based on the concept of consistent interpolation for transverse displacements and flexural rotations to avoid transverse shear locking as discussed in the 2nd paragraph, the torsional rotation and warping function are consistently interpolated to include the influence of warping shear stress.

Considering that distribution of bending rotations or warping function along the beam length is non-linear when a thin-walled beam is under non-uniform bend or torsion, an interior node is intended to be added to a thin-walled beam element to improve the analytical precision [52]. In 1970s, interior nodes were initially introduced to the two-dimensional incompatible elements proposed by Wilson et al. [53] and Herrmann [54] for improvements in accuracy. Celia and Gray [55] concluded that interior nodes positioned such that they do not affect the Jacobian of the coordinate transformation as well as side nodes at the same relative distance from corner nodes in both local and global space provided improved accuracy. Gupta et al. [56] adopted a shell element interpolated by cubic B-splines in the analysis, in which DOFs at the interior nodes of the meridian are two nodal displacements while DOFs at the edge nodes include two displacements and the rotation of the meridian. Houmat [57] presented a four-node Timoshenko beam element (with two end nodes and two interior nodes) for the vibration analysis, whose displacement fields were described by a cubic polynomial and a variable number of trigonometric sine terms. Malsch and Dasgupta [58] constructed new test functions which made it possible that the interior nodes could be located at any desired position of the element. Ho and Yeh [52] derived a family of enriched elements to improve their accuracy by introducing bubble functions and interior nodes of Lagrange elements. Krishnan [59] developed an efficient beam element with four interior nodes, which is composed of three fiber segments and two elastic segments to capture the overall features of the elastic and inelastic responses of slender columns and braces. Tsai [60] proposed a floating interior-node scheme to eliminate the null modes of flexural vibration in the Timoshenko frames. To date, beam elements with interior nodes, however, have not yet been found in the documents to be applied to the analysis of thin-walled members.

In this paper, a shear-locking free spatial beam element with closed cross-section is proposed based on the authors' existing research work [61–64] on the open thin-walled beams. The influence of flexural and torsional coupling is considered on the basis of the kinematic description of a general cross-section and transverse displacements and bending rotations are consistently interpolated to avoid shear locking. To include the influence of warping shear stress, the consistent interpolation is also adopted in the discretization of the torsional rotation and warping function. With the view of improvement in the element accuracy, an interior node is introduced to the thin-walled beam element and cubic Hermitian interpolation polynomials are adopted for transverse displacements and torsional rotation while quadratic Lagrangian interpolation polynomials are adopted for bending rotations and warping function. It can be found that the consistent interpolation approach adopted in this paper is different from and more accurate than that in the documents [13,28], in which only quadratic and linear Lagrangian interpolation polynomials are employed for transverse displacements and bending rotations, respectively. The nodal displacement vector of the proposed element is divided into two components. One is the external displacement vector (Eq. (38)) related to the inter-element displacement compatibility and the other is associated with the internal DOFs that are unrelated to the displacement compatibility (Eq. (39)). It should be noted that the internal DOFs in the proposed beam element are different from those in the documents [52–60] including only DOFs of the interior nodes. They consist of two parts. One is composed of DOFs corresponding to the flexural rotations and warping function of the interior node and the other is composed of the first derivative of the transverse displacements and the torsional rotation at both end nodes (please see Eq. (39)). The element stiffness matrix is derived according to the classical variational principle and the internal DOFs of the element are condensed in the equilibrium equations to reduce the number of the total DOFs. Several examples are analyzed with the proposed element, and results are compared with theoretical and numerical solutions to verify the accuracy and convergence property of the proposed finite beam element.

In the following sections, the basic assumption, strains and stress resultants, shape function matrix, element stiffness matrix, transformation matrix on the proposed finite element, the numerical examples and conclusions will be discussed.

2. Assumptions

The present research is confined to the analysis of a thin-walled straight beam with closed cross-section. The material is elastic in