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Experimental investigations of size effect in reinforced concrete beams failing by shear

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ABSTRACT

Laboratory tests were carried out on concrete beams with longitudinal bars and without shear reinforcement. Slender concrete beams with steel bars were subjected to four-point bending with the ratio of the shear span to the effective depth equal to $\alpha/D = 3.0$ while short concrete beams with basalt bars were subjected to three-point bending with $\alpha/D = 1.0$. The beams were geometrically similar. Load–deflection curves and cracks were registered. In addition, the Digital Image Correlation (DIC) technique was used to measure displacements on the outer surface of concrete to visualize localized zones. A pronounced size effect was measured in concrete beams.

1. Introduction

A size effect in concrete elements denotes that both the nominal structural strength and material brittleness (ratio between the energy consumed during the loading process after and before the stress–strain peak) always decrease with increasing element size under tension [1–3]. Thus, the concrete elements become ductile on a small scale and perfectly brittle on a sufficiently large scale. Two size effects are of a major importance: energetic (or deterministic) and statistical (or stochastic) one. A deterministic size effect is caused by the formation of a region of intense strain localization with a certain volume (micro-crack region – called also fracture process zone FPZ) which always precedes discrete macro-cracks. Strain localization volume is not negligible to the cross-section dimensions and is large enough to cause significant stress redistribution in the structure and associated energy release. The specimen strength increases with increasing ratio l_c/D (l_c – characteristic length of the micro-structure influencing both the size and spacing of localized zones, D – characteristic structure size). The nominal structural strength which is sensitive to the size of FPZ as compared to the specimen size cannot be appropriately estimated in laboratory tests, since it is different for various specimen sizes. In turn, a statistical (stochastic) effect is caused by the spatial variability/randomness of the local material strength. The first statistical theory has been introduced by Weibull [4] (called also the weakest link theory) which postulates that a structure is as strong as its weakest component. The structure fails when its strength is exceeded in the weakest spot, since stress redistribution is not considered. A combination of the energetic theory with the Weibull statistical theory led to a general energetic-statistical theory [5]. The deterministic size effect occurs for not too large structures and the Weibull statistical size effect is obtained as the asymptotic limit for very large structures. The deterministic size effect can also occur in reinforced concrete beams without shear reinforcement wherein diagonal shear and tensile fracture takes place in concrete. The diagonal cracks have essentially similar paths and relative lengths at the maximum load independently of the beam size. Therefore, this size effect can be described by the analytical size effect law (SEL) of Type II according to Bazant [1,6–8] for structures of a positive similar geometry possessing notches or large stress-free cracks that grow in a stable manner up to the maximum load (it is assumed that fracture energy is constant and size-independent) (Fig. 1):

$$\sigma_N(D) = \frac{Bf_t}{\sqrt{1+\frac{D}{D_o}}}, \qquad (1)$$

where the parameter f_t denotes the tensile strength, B is the dimensionless geometry-dependent parameter (depending on the geometry of the structure and crack) and D_o denotes the size-dependent parameter called transitional size (both unknown parameters to be determined by fitting Eq. (1) to the experimental

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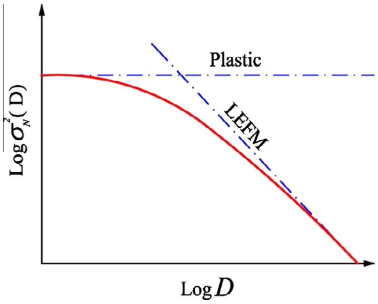

Fig. 1. Size effect law by Bazant [3] in logarithmic scale with σ_n – nominal strength, D – characteristic structure size for notched structures (material strength is bound for small sizes by plasticity limit, whereas for large sizes the material follows Linear Elastic Fracture Mechanics LEFM).

data). The law approaches 0 for $D \to \infty$. For small structures ($D \to 0$), the size effect disappears. Thus, the size effect is strong only in a limited size range.

The intention of our paper is to show the experimental results of a size effect in both slender and short concrete beams without shear reinforcement failing by shear under bending. The numerical evaluation results of our experiments based on a stochastic enhanced elasto-plasticity [9] will be published in the next paper. Comprehensive tests were carried out with slender concrete beams including steel bars and subjected to four-point bending. In addition, some initial tests were carried out with single short concrete beams including basalt bars and subjected to three-point bending in order to check the effect of a different possible failure mode by decreasing both the beam slenderness and the reinforcement stiffness. Special attention was paid to the detailed description of a fracture process (inherently connected to a size effect) by determining the height, width and spacing of localized zones. The height and width of localized zones were determined with the aid of the digital image correlation (DIC) technique, which allowed for obtaining a displacement field on the concrete surface (through that a failure mode might be precisely determined). All beams were geometrically similar. The innovative points of our investigations are following: (a) comparative size effect tests in slender and short concrete beams with steel and basalt longitudinal reinforcement, (b) an application of DIC to measure the width and height of localized zones in longitudinally reinforced concrete beams and (c) a detailed description of a failure mechanism in reinforced concrete beams.

Table 1
Geometry of slender RC beams (H – beam height, L – beam length).

Dimension	Test series		
	SL20	SL40	SL80
H (mm)	200	400	800
L (mm)	1500	3000	6000

The outline of the present paper is as follows. First, after the introduction (Section 1) and literature review (Section 2), the experimental set-up is described in Section 3. The experimental results on a size effect are described and discussed in Section 4. Conclusions are listed in Section 5.

2. Literature review

A size effect on the shear strength of reinforced concrete (RC) beams without stirrups was confirmed in experiments among others by: Leonhardt and Walther [10], Kani [11,12], Bhal [13], Taylor [14], Chana [15], Walraven [16], Iguro et al. [17], Bazant and Kazemi [18], Shioya and Akiyama [19], Kim and Park [20], Grimm [21], Ghannoum [22], Kawano and Watanabe [23], Podgorniak-Stanik [24], Yoshida [25], Angelakos et al. [26] and Lubell et al. [27].

In the experiments by Leonhardt and Walther [10], the size of reinforced concrete beams was in a proportion 1:2:3:4 ($\alpha/D = 3$). The reinforcement was composed always of 2 steel bars. The reinforcement ratio was $\rho = 1.35\%$. The bearing capacity of beams decreased with increasing beam size. Kani [11,12] investigated varying beam depths D, longitudinal steel percentages ρ and shear span-to-effective depth ratios α/D. The value of $a/D = 2.5$ was a transitional point between failure modes independently of D and ρ. In the beams with $a/D \leqslant 2.5$ inclined cracks first developed and after a redistribution of internal forces, the beams were able to carry the additional load due to an arch action. For $a/D > 2.5$, the failure was sudden and brittle after the first diagonal cracks appeared. According to Taylor [14], a size effect could be nearly eliminated with a proper geometric scaling of all beam components including the cover thickness, reinforcement layout and maximum aggregate size. In the tests by Chana [15] with RC beams of the depth 150–750 mm with $\rho = 0.74–0.83\%$ and $a/D = 3$, a pronounced size effect occurred in both lightweight and normal concrete. Bazant and Kazemi [18] performed tests on geometrically similar beams with a size range of 1:16 ($D = 25–406$ mm, $a/D = 3$ and $\rho = 1.65\%$). The smallest specimens failed in flexure, while the others in diagonal shear. Kim and Park [20] carried out tests on high

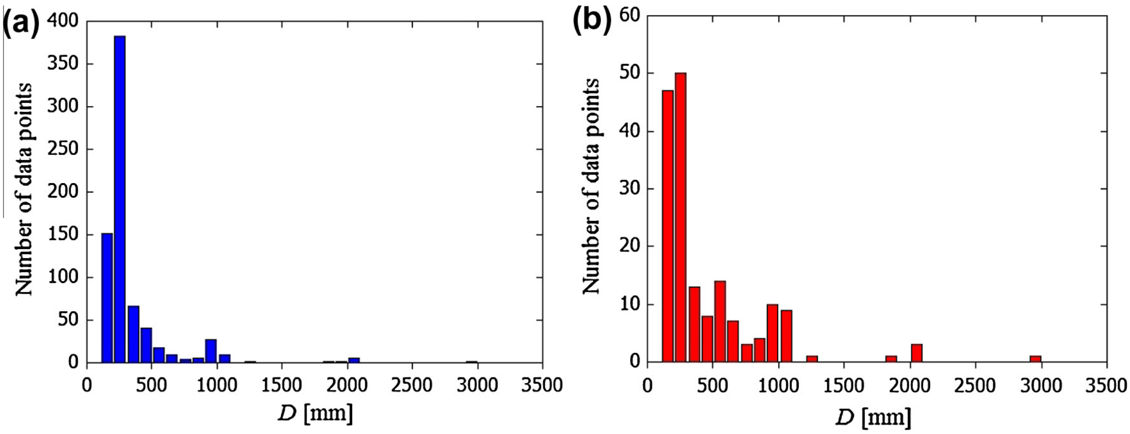

Fig. 2. Histogram of beam depths in the extended shear database on size effect: (a) all size effect experimental data points and (b) selected experimental data points from size effect tests [30].