Engineering Structures 52 (2013) 697–706

Contents lists available at SciVerse ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Elastic out-of-plane buckling load of circular steel tubular truss arches incorporating shearing effects

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ARTICLE INFO

Article history:
Received 6 December 2012
Revised 16 March 2013
Accepted 19 March 2013
Available online 25 April 2013

Keywords:
Buckling
Out-of-plane
Truss
Arch
Shear rigidity
Torsional rigidity
Shear deformation

ABSTRACT

For steel tubular truss arches, calculations of sectional rigidity especially the torsional rigidity, as well as the effect of shear deformation on out-of-plane buckling are not available yet. This paper investigates the sectional rigidities of trusses and the out-of-plane buckling loads of pin-ended circular steel tubular truss arches in uniform axial compression and in uniform bending. Firstly the compression rigidity, flexural rigidity, shear rigidity and torsional rigidity of latticed configuration for trusses are deduced. Then the out-of-plane buckling equations for circular monosymmetric arches incorporating the effect of shear deformation are established using a static equilibrium approach. Lastly the closed form solutions for out-of-plane buckling loads of pin-ended circular truss arches in uniform compression and in uniform bending are obtained. It is found that the axial deformation of chord tubes need be taken into account in the derivation of shear rigidity and torsional rigidity of latticed configuration. Due to the curved arch profile, the effect of shear deformation on out-of-plane buckling loads of truss arches is much smaller than that on straight truss columns.

1. Introduction

Steel arches have high in-plane load-carrying capacity. Compared with solid-web steel arches, steel tubular truss arches that resist external loading by the curved profile and latticed configuration, are more widely used in large span spatial structures such as stadium buildings and steel bridges due to their favorable architectural effect and higher load-carrying efficiency. It is known that a circular arch under in-plane loading may buckle in the plane of loading in a bifurcation mode or in a limit point instability mode [1–7], and it may also suddenly displace laterally and twist out of its plane of loading and fail in a lateral-torsional buckling mode [1,2,8–22].

Because the lateral and torsional stiffnesses of an arch are usually much smaller than its in-plane stiffnesses, when the arch does not have adequate lateral bracings, it may fail in a flexural–torsional mode before its in-plane failure. As a result, attention should be paid to the out-of-plane stability of steel tubular truss arches for practical applications. Before investigations for the inelastic strength design of arches under general loading by a combination of axial compression and bending actions, the linear elastic out-of-plane buckling loads of circular arches in uniform compression and in uniform bending need be studied. Substantial investigations [8–22] have been carried out mainly into the out-of-plane stability of solid-web circular steel arches since the early contributions by Timoshenko and Gere [1] and Vlasov [2], among which either static equilibrium approaches [1,2,19–22] or energy approaches [8–19] were adopted. Timoshenko and Gere [1] derived the out-of-plan buckling loads of circular arches with narrow rectangular section, and discussed the effect of loading direction. Vlasov [2] obtained the buckling loads of circular arches with open cross-section, by introducing an analogy between the curved member and a straight beam–column. Yoo [9] made a similar analogy by a virtual work approach for the buckling of circular arches, which was pointed out to produce some errors by [10–14,17,19–21]. Dou and Guo [22] presented closed form solutions of buckling loads for doubly symmetric circular arches by a static equilibrium approach, which were exactly the same with those derived by Pi and Bradford [19] using an energy approach. However, the closed form solutions for elastic out-of-plane buckling loads of circular arches in uniform compression and in uniform bending obtained by different researchers have some discrepancies between each other, which as explained by Pi and Bradford [19], are mainly because different expressions for nonlinear components of the longitudinal normal strain were used during the energy approach derivations. Meanwhile they also pointed out [19], if the linear elastic out-of-plane buckling of circular arches was concerned, the static equilibrium approach still supplies an effective way to obtain the accurate buckling loads.

Due to the latticed configuration, stability theories of steel tubular truss arches are more complex than those of solid-web

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http://dx.doi.org/10.1016/j.engstruct.2013.03.030

Notation

A_c	cross-sectional area of a single chord tube
A_d	cross-sectional area of a single diagonal tube
A_t	cross-sectional area of a single transverse tube
a, b	side lengths of cross-section
D_c	external diameter of a chord tube
D_d	external diameter of a diagonal tube
D_t	external diameter of a transverse tube
d_{ci}	distance from the axis of chord tube i to the centroid of entire cross-section
E	modulus of elasticity
EA	compression rigidity of cross-section
EI_w	warping rigidity of cross-section
EI_y	flexural rigidity of cross-section
f	rise of an arch
G	shear modulus of elasticity
GJ	torsional rigidity of cross-section
i_0	polar gyration radius of cross-section
K_v, K_{vi}	shear rigidity
K_{vx}, K_{vy}	shear rigidities along the x, y axes
L	span of an arch
L_c	length of a chord tube within a panel of a truss
L_d, L_{di}	length of a diagonal tube
L_t, L_{ti}	length of a transverse tube
M_{cr}	buckling moment of an arch
M_x, M_y	bending moments about the x, y axes
M_z	torsional moment over cross-sections
m_{zq}	distributed external torque about the shear center of cross-section
N	axial force of cross-section
N_c, N_{ci}	axial force of a chord tube
N_{cr}	buckling axial force of an arch
N_d, N_{di}	axial force of a diagonal tube
N_t, N_{ti}	axial force of a transverse tube
P_y	the first mode flexural buckling load of an axially-loaded pin-ended column
P_z	the first mode torsional buckling load of an axially-loaded pin-ended column
Q_x, Q_y	shearing force along the x, y axes
q_x	component of external distributed load along the x axis
R	radius of an arch
r_{ci}	distance from the sectional centroid of chord tube i to the shear center of entire cross-section
r_i	distance between the torsional center and side i of cross-section
S	developed length of cross-sectional centroid axis of an arch
t_c	wall thickness of a chord tube
t_d	wall thickness of a diagonal tube
t_t	wall thickness of a transverse tube
u	out-of-plane translational displacement of cross-sectional centroid of an arch
V, V_i	shearing force
y_q	loading position of external force over cross-section along the y axis
y_s	coordinate of the shear center over cross-section along the y axis
α_i	included angle between the direction of external shearing force and surface i of a truss
β	out-of-plane rotation of cross-section of an arch
β_y	coefficient of sectional asymmetry
γ, γ_i	shear angle
γ_x, γ_y	shear angles along the x, y axes
θ, θ_i	included angle between a diagonal tube and a chord tube
κ_z	unit twist rotation of cross-section of an arch about the z axis
ρ_x, ρ_y	radiuses of curvatures about the x, y axes induced by bending moments
$\rho_{x\gamma}$, $\rho_{y\gamma}$	radiuses of curvatures about the x, y axes incorporating the effect of shear deformation
ψ	coordinate of central angle of an arch
ω	twist rotation of cross-section of an arch
Δ_c	deformation of a chord tube
Δ_t, Δ_{ti}	deformation of a transverse tube
Θ	subtended angle of an arch

arches. The differences between them mainly include two aspects, of which one is to calculate the sectional rigidities such as shear rigidity and torsional rigidity of trusses, the other is to incorporate the effect of shear deformation of trusses in the buckling loads. However, up to now none of the problems above is well solved for out-of-plane buckling of steel tubular truss arches.

As a beginning of investigations into the out-of-plane stability design under general loading, this paper focuses on the linear elastic out-of-plane overall buckling loads of pin-ended circular steel tubular truss arches subjected to uniform compression and in-plane uniform bending (Fig. 1). Firstly, the compression rigidity, flexural rigidity, shear rigidity and torsional rigidity for trusses are deduced theoretically. Secondly the out-of-plane buckling equations for circular monosymmetric arches incorporating the effect of shear deformation are established via a static equilibrium approach, which are suitable for both solid-web arches and tubular truss arches. Then the closed form solutions for out-of-plane buckling loads of pin-ended circular truss arches in uniform compression and in uniform bending are presented respectively, with consideration of the shear effects of latticed configuration, loading position over cross-section and sectional asymmetry.

For circular arches in uniform compression (Fig. 2), the distributed radial loads are classified as three types, namely hydrostatic load, load directed towards a fixed point and load with fixed direction [19,21,22], among which the last one results in the lowest out-of-plane buckling load. In this paper, the radial load with fixed direction is involved. The geometry of the truss arches studied is shown in Fig. 1. Θ is the subtended angle of the arch, ψ is the coordinate of central angle, R is the radius of the arch (distance from the cross-sectional centroid to the center of circle arc) and S is the developed length of cross-sectional centroid axis. The origin point o of coordinate system o-xyz is located at the centroid of cross-section. The web members of steel tubular truss arches studied in this paper are classified as two groups, of which the first ones are perpendicular to the chord tubes named as the transverse tubes, the other ones are oblique and named as the diagonal tubes, as shown in Fig. 1. Also, the truss arches are classified as two types according to the arrangement of web members. One type is Pratt truss characterized by having its diagonal tubes all slanted down towards one direction, and the other is Warren truss of which the diagonal tubes constitute a series of isosceles triangles.

2. Finite element model description

Finite element program ANSYS 13.0 [24] is adopted to perform linear elastic analyses for sectional rigidities of trusses, as well as numerical enginbuckling analyses for elastic out-to-plane buckling