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Investigation of computational and accuracy issues in POD-based reduced order modeling of dynamic structural systems

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ABSTRACT

In this paper, we investigate the performance of reduced order modeling of dynamic structural systems based on the proper orthogonal decomposition (POD) technique. Singular value decomposition of the so-called snapshot matrix is adopted to generate the reduced space, onto which the system equations of motion are projected to speedup the computations.

To get insights into the achievable speedup and the capability of POD to provide an input-independent reduced model, we consider the 39-story Pirelli tower in Milan-Italy. First, we assume that a shear model of the building is excited by the May 18-1940, M_w 7.1, El Centro earthquake, and generate the data ensemble necessary to build the reduced model. Second, we assess the local and global accuracies of the same reduced model in tracking the dynamics of the building, if excited by the May 6-1976, M_w 6.4, Friuli earthquake and by the January 17-1995, M_w 6.8, Kobe earthquake, which differ from the El Centro one in terms of excited vibration frequencies. We show that POD allows to attain a speedup approaching 250, when the reduced order model is asked to feature a high accuracy; moreover, POD tends to outperform a standard modal analysis at increasing number of modes retained in the model.

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1. Introduction

Accurate, low-dimensional reduced order models are of primary importance for large scale structures [1,2]. To speedup prognostic analyses forecasting the effects of extreme loadings (like, e.g., earthquakes or low-velocity impacts) leading to local or global failure mechanisms [3–5], or to enhance the engine of structural health monitoring procedures (defined as *enabling technologies* of a value chain in [6]), fast computational tools are required to provide the response to the external loads of *linear comparison* models of the structures [7–10].

If nonlinearities are heavily affecting the structural response, multi-scale [11,12] and domain-decomposition [13] approaches need to be resorted to reduce the computational burden of numerical simulations, still keeping a target accuracy in critical regions where localized failures are expected to take place [3,5]. With a slightly different target, we assume here that nonlinearities are well confined in small portions of a structure, whereas the remaining part behaves linearly. We therefore focus on this latter substructure and investigate an approach to reduce the order of dynamically excited systems, which is based on the proper orthogonal decomposition (POD) methodology [14–16]. Even if this topic is not addressed in this paper, we also assume that local failure mechanisms causing drifts in the structural response, can be simultaneously assessed through ad-hoc structural health monitoring systems.

Instead of being routed by physical reasons (e.g. coarse graining away from the regions behaving nonlinearly), POD turns out to be governed by purely mathematical tools. POD has been developed independently in different research fields with different names, like, e.g. principal component analysis [17], singular value decomposition [18], Karhunen-Loeve decomposition [19,20]; it was proved that all these algorithms are different variants of POD, see [21].

POD automatically looks for a dependence structure among the degrees of freedom (DOFs) of a space discretized system, which are normally assumed to be independent. The goal is achieved through a set of ordered orthonormal bases, or proper orthogonal modes, POMs (to be distinguished from the standard vibration modes of the structure). They are obtained through a singular value decomposition of the so-called snapshot matrix [22], which collects snapshots of the structural response to the loading during an initial stage of training of the analysis. By way of an energy-based accuracy index (to be discussed in Section 3), POD also provides a rationale to define the number of POMs to be retained in the reduced order model (ROM). Some authors recognized a few weaknesses in such methodology, see e.g. [23–25]: the training of POD requires the full model to be run in the initial stage of computations, hence

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the speedup of the analysis is detrimentally affected; the duration of the training stage cannot be a priori defined, since it can be judged only through convergence (granted by the linearity of the analyzed systems) of the POD subspace, wherein the ROM is thereafter allowed to evolve; since training is obtained under specific loading conditions, it is not simple to assure that the accuracy of the ROM does not change under different loadings. Moreover, there is no consensus on how to attack nonlinear problems with the same procedure, or similar ones [9,26,27]; an approach, obtained by coupling POD for a linearized model of the structure and Kalman filter for adapting on-the-fly the ROM when nonlinearities come into play, was proposed in [28]. This issue in not discussed any further here below, where focus is on the accuracy, efficiency and robustness features of POD-based ROMs for dynamics structural systems.

To assess the computational gain offered by POD, we consider the case of tall buildings excited by earthquakes. In the study we handle a numerical model of a 39-story building located in Milan-Italy, the Pirelli tower; under horizontal actions, we adopt for it a shear-type deformation model. Even if an automatic update of POMs has not been adopted here, we show that a heuristic approach, which consists in a training stage covering a fundamental period of vibration of the building, allows to attain convergence of the shape of the POMs themselves. POD effectively scales down the computational costs by up to two-three orders of magnitude, and provides at least the same overall accuracy of modal analysis-based ROMs; by increasing the required energy-based accuracy of the ROM, POD progressively outperforms modal analysis. We also show that ROMs of the building at varying target accuracy, all built with snapshots of the response to the May 18-1940 El Centro earthquake, can be adopted also under different loading conditions, like e.g. earthquake records featuring different frequency contents. Through the reported critical analysis, we therefore provide a rational theoretical frame for real-time monitoring of the health of structures undergoing localized damage, as shown in [28].

The remainder of the paper is organized as follows. Section 2 provides a few details about the explicit time integration scheme adopted to advance in time the solution of the structural equations of motion; a discussion on how the relevant conditional algorithmic stability is managed in the full and reduced order analyses is presented as well. Section 3 gathers fundamentals of POD and singular value decomposition, with an explanation on how accuracy of the reduced model can be a priori set. To assess the quality of the solutions provided by POD, even through comparison with modal analysis, in Section 4 results are shown for the mentioned 39-story building excited by three different earthquakes: the May 18-1940, M_w 7.1, El Centro one; the May 6-1976, M_w 6.4, Friuli one; and the January 17-1995, M_w 6.8, Kobe one. Finally, Section 5 presents some concluding remarks on the present work, and possible future activities to extend the applicability of the studied methodology to the health monitoring of real structures.

2. Structural dynamics and time integration

Let the dynamic response of the structural system to the external loads be described by the following linear equations of motion:

$$\boldsymbol{M}\ddot{\boldsymbol{u}}(t) + \boldsymbol{D}\dot{\boldsymbol{u}}(t) + \boldsymbol{K}\boldsymbol{u}(t) = \boldsymbol{F}(t) \tag{1}$$

where $\boldsymbol{M}$ is the mass matrix; $\boldsymbol{D}$ is the viscous damping matrix; $\boldsymbol{K}$ is the stiffness matrix; $\boldsymbol{F}$ is the time-dependent external force vector; $\ddot{\boldsymbol{u}}, \dot{\boldsymbol{u}}$ and $\boldsymbol{u}$ are the time-varying vectors of system accelerations, velocities and displacements, respectively. As for a shear model of buildings (like the one to be adopted in Section 4), these vectors gather the lateral displacements, velocities and accelerations of the storys.

Eq. (1) is usually arrived at once the structural system has been space discretized (e.g. through finite elements), or once assumptions concerning the behavior of the building (e.g. shear-type deformation) have been taken into account. This preliminary stage of the analysis can affect the sparsity of matrices in (1), and can therefore have an impact on the speedup obtained through POD as well.

The solution of the vectorial differential Eq. (1) is here advanced in time by making use of the Newmark explicit integration scheme. We subdivide the time interval of interest according to $[t_0\ t_N] = \bigcup_{i=0}^{N-1}[t_i\ t_{i+1}], N$ being the number of time steps featuring constant size $\Delta t = t_{i+1} - t_i$; within $[t_i\ t_{i+1}]$, the time marching algorithm can be partitioned according to [29]:

- predictor stage:

$$\begin{aligned}\tilde{\boldsymbol{u}}_{i+1} &= \boldsymbol{u}_i + \Delta t\dot{\boldsymbol{u}}_i + \Delta t^2\left(\frac{1}{2} - \beta\right)\ddot{\boldsymbol{u}}_i \\ \dot{\tilde{\boldsymbol{u}}}_{i+1} &= \dot{\boldsymbol{u}}_i + \Delta t(1-\gamma)\ddot{\boldsymbol{u}}_i\end{aligned} \tag{2}$$

- explicit integrator stage:

$$\ddot{\boldsymbol{u}}_{i+1} = \boldsymbol{M}^{-1}(\boldsymbol{F}_{i+1} - \boldsymbol{D}\dot{\tilde{\boldsymbol{u}}}_{i+1} - \boldsymbol{K}\tilde{\boldsymbol{u}}_{i+1}) \tag{3}$$

- corrector stage:

$$\begin{aligned}\boldsymbol{u}_{i+1} &= \tilde{\boldsymbol{u}}_{i+1} + \Delta t^2\beta\ddot{\boldsymbol{u}}_{i+1} \\ \dot{\boldsymbol{u}}_{i+1} &= \dot{\tilde{\boldsymbol{u}}}_{i+1} + \Delta t\gamma\ddot{\boldsymbol{u}}_{i+1}\end{aligned} \tag{4}$$

The explicit integrator (3) allows to easily extend the present results to the nonlinear regime, if the elastic term $\boldsymbol{Ku}$ in (1) is replaced by a possibly path- and history-dependent internal force vector term $\boldsymbol{F}^*(\boldsymbol{u})$ (see, e.g. [30]).

To provide exemplary results concerning the performance of POD, in the simulations here collected we have adopted $\beta = 0$ and $\gamma = 0.5$, hence resorting to the central difference algorithm; similar results can be obviously obtained with different values of the algorithmic parameters β and γ.

Since the above algorithm is conditionally stable, Δt needs to be upper bounded by [29,31]:

$$\Delta t^{cr} = \frac{T_n}{\pi} \tag{5}$$

where T_n is the period of vibration associated with the highest oscillation frequency in the numerical model. It may result that the critical time step size can be increased in the ROM, since high frequency oscillations are automatically filtered out of the relevant numerical solution. Here we purposely avoid to take advantage of this effect, and assume $\Delta t_{\mathrm{POD}} = \Delta t$; hence, the provided speedups are linked to the reduction of the number of handled DOFs only, and need to be considered as a lower bound on the achievable ones. Additional results, highlighting also the effects of $\Delta t_{\mathrm{POD}} > \Delta t$ (still assuring $\Delta t_{\mathrm{POD}} < \Delta t^{cr}_{\mathrm{POD}}$ in the ROM) on the costs and accuracy of the solution, will be reported in a future work specifically focused on a model problem (see also [32]).

3. Fundamentals of proper orthogonal decomposition for dynamic structural systems

The aim of reduced order modeling is to automatically find a solution to the following two conflicting requirements: create the smallest possible numerical model of the original dynamic system; preserve accuracy in the description of the system behavior. Standard techniques try to extract fundamental features from the dynamic model, so as the governing equations can be thereafter projected onto a reduced state space, or subspace.

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