Engineering Structures 54 (2013) 180–191

Contents lists available at SciVerse ScienceDirect

Engineering Structures

journal homepage: www.elsevier.com/locate/engstruct

Assessing small failure probability by importance splitting method and its application to wind turbine extreme response prediction

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ARTICLE INFO

Article history:
Received 7 January 2013
Revised 28 February 2013
Accepted 28 March 2013
Available online 15 May 2013

Keywords:
Failure probability
Multi-correlated excitations
Importance splitting method
Multivariate autoregressive model
Monte Carlo simulation
Extreme response
Wind turbines

ABSTRACT

This study presents an effective simulation framework with importance splitting (ISp) method for estimating small failure probabilities of dynamic structures with multi-correlated stochastic excitations, and addresses its application to predictions of large wind turbine extreme responses. The ISp method, also referred to as subset simulation with splitting, splits important sample paths into multiple branches at various stages in the simulation. It permits the estimation of a small failure probability of a rare event through estimations of conditional probabilities of intermediate subset events. The framework presented in this study combines the ISp method with multivariate autoregressive (MAR) modeling of stochastic excitations. The MAR model of excitations is established based on their cross power spectral density matrix, which transfers the stochastic excitations as the output of a loading system with a vector-valued uncorrelated white noise process as input. This scheme is very efficient in generating offsprings of loading and response time histories conditional on the intermediate events with very low rejection rate, which facilities the application of ISp method to different kinds of stochastic single and multiple excitations. The effectiveness and accuracy of the proposed new scheme are verified by a reliability problem of earthquake-excited 5-story building, and by the estimation of extreme responses of a 5 MW onshore wind turbine with very small exceeding probabilities. Finally, this framework is applied to validate the extrapolation procedure of estimating wind turbine long-term extreme responses with various mean recurrence intervals from short-term simulations of turbine response histories, which is mandated by current wind turbine design standards.

1. Introduction

Reliability- and performance-based wind turbine design requires estimation of extreme responses with various mean recurrence intervals (MRIs), say 50 years. The International Electrotechnical Commission standard [21] recommends a load (response) extrapolation procedure to estimate the large extreme responses from short-term 10 min simulations of turbine response at various mean wind speeds. Based on these simulations, the distributions of extreme responses over 10 min conditional on various wind speeds are estimated and fitted with given probability distribution models, which are then combined with the distribution of mean wind speed for the estimation of overall extreme distribution. The extreme responses with various MRIs are then estimated from this short-term extreme value distribution. For instance, the 50-year extreme response corresponds to an exceeding probability of 3.8×10^{-7} in terms of 10 min maximum, i.e., requires information of probability distribution at very upper tail. Previous studies have shown that the predicted long-term extreme responses are very sensitive to the probability models used in fitting the extreme value distributions at various wind speeds. Different distribution models, while fitting the simulated extreme data well, can have very distinct behaviors in the upper tails that result in very different predictions of long-term extremes [1,14].

In order to provide a guideline for the appropriate implementation of the extrapolation procedure mandated by current wind turbine design standards, an international working group was organized and a number of studies were carried out. Among these research efforts, Moriarty [30] carried out direct Monte Carlo Simulation (MCS) of turbine response for a total of 5 years of operation involving about 48,000 samples of 10 min simulations per year. Recently, Natarajan and Verelst [33] utilized principal component analysis technique to weight the sampled outliers and reduce the extrapolation uncertainty from different sample sizes. Sichani et al. [39] used the asymptotical sampling technique to evaluate the first passage failure probabilities of wind turbines by extrapolation of the safety index. Sichani et al. [41] utilized the extrapolation of the average conditional exceedance rate (ACER) [32] for the estimation of the first passage failure probabilities of wind turbines. The application of Markov chain Monte Carlo simulation of turbine extreme response was presented by Sichani and Nielsen

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http://dx.doi.org/10.1016/j.engstruct.2013.03.051

[42] with challenges for high dimensional multi-correlated excitations in the generation of conditional samples by Markov chain.

The standard (direct) MCS method is well known as a robust tool in dealing with high dimensional reliability problems of complicated structural systems. However, this method is not suitable for generating large extremes with small exceeding probabilities, because of huge number of samples needed. The required sample size of MCS for a rare event is inversely proportional to its occurrence probability. Therefore, use of variance reduction approaches becomes essential to enhance the efficiency of simulations of a rare event. A number of studies have given a comprehensive review on various variance reduction approaches as applied to structural dynamic problems (e.g., [35,28,27]).

The first class of variance reduction Monte Carlo (VRMC) simulation is importance sampling (IS). This method forces the important events sampled more frequently than direct MCS by introducing a so-called 'importance sampling' probability density function (PDF). The IS method has been widely applied to time invariant problems, and its applications to dynamic problems have been limited due to the difficulties in selecting adequate importance sampling PDF (e.g., [28,4,27,34]). Besides, the line sampling (LS) and asymptotic sampling (AS) approaches have also been utilized to reliability analysis of dynamic structures (e.g., [25,6,40]). The LS method employs lines instead of points in the random parameter space in order to collect information of the failure probability. However, the efficiency and accuracy of its application to high dimensional problems are ensured only for the cases where a dominant direction towards the failure domain could be evaluated [25,36]. On the other hand, the AS method is an approach by changing excitation level to cause more out-crossing of the barrier and then adjust the estimated probabilities, and to use the advantage of the asymptotic behavior of the safety index for multi-normal probability integrals associated with an extrapolation technique. However, this method is unsuitable for failure problems defined in terms of a finite failure domain (an 'island of failure') and is still based on extrapolation from limited information [6,40].

The subset simulation (SS) approach seems attractive in dealing with small probability evaluation problems [3], which has been applied to dynamic reliability analysis of earthquake-excited structures [5]. The principle idea of the SS approach is to transform the simulation of a rare event into the simulations of a sequence of successive intermediate events with larger probabilities. The small probability of the rare event can be estimated as a product of larger conditional probabilities of the intermediate events. The key to the implementation of SS approach is generation of the conditional samples (offspring samples) for the quantification of conditional probabilities. The general Markov chain Monte Carlo (MCMC) algorithm was used in Au and Beck [3,5], referred to as SS/MCMC. Another form of SS approach as applied to first passage problems of deterministic structural systems was introduced by Ching et al. [10], using the concept of response trajectory splitting, referred to as subset simulation with splitting (SS/S). A hybrid scheme combining MCMC and splitting was introduced in Ching et al. [11], which can be used to evaluate first passage problems of uncertain and deterministic structural systems.

The SS/S has also been referred to as importance splitting (ISp) method, and has a long history of development as a classical comparative technique in rare event simulation. The first usage of this concept in simulation is close to the description by Kahn and Harris [24]. A more recent version of the splitting method was introduced by Villén-Altamirano and Villén-Altamirano [46], named as RESTART. A detailed analysis of the mathematical properties for multi-level splitting can be found in Glasserman et al. [18,19] and Garvels [16]. All these research efforts have laid a solid theoretical foundation for understanding its performance and applications to various simulation problems. Basically, there are two requirements for the splitting method. One is that the splitting point should include all the previous information, in other words, the process should be assumed as Markov process. Since most of the structure systems are causal, the Markov property is not really a limitation for the proper use of this method. Another requirement is that the most likely path to a rare event cannot differ too greatly from the most likely path to an intermediate level. Otherwise the estimator appears biased with high probability even for large sample size [18]. Fortunately, this requirement is always accorded with in structural engineering if the system is not extremely very nonlinear [45].

In this study, an effective simulation framework with ISp (or SS/S) method is presented for estimating small failure probabilities of dynamic structures with multi-correlated stochastic excitations. The framework combines the ISp method with multivariate autoregressive (MAR) modeling of stochastic excitations. The MAR model of excitations is established based on their cross power spectral density matrix, which transfers the stochastic excitations as the output of a loading system with a vector-valued uncorrelated white noise process as input. This scheme is very efficient in generating offspring of loading and response time histories conditional on the intermediate events with very low rejection rate, which also facilities the application of ISp to different kinds of stochastic single and multiple excitations. The optimizations of the parameters involved in this method for improving its performance are also discussed. The effectiveness and accuracy of the proposed new scheme are verified by a reliability problem of earthquake-excited 5-story building, and by the estimation of extreme responses of a 5 MW wind turbine with very small exceeding probabilities. Finally, this framework is applied to evaluate extreme responses of wind turbines with MRI of 50 years, which is required by current wind turbine design standards. The results are verified with the multi-year simulation data documented in Moriarty [30], and provide new insights on extrapolations of extreme responses.

2. Importance splitting method with MAR modeling of excitations

2.1. Implementation of the ISp method

Consider the dynamic reliability problem of a deterministic structural system under stochastic excitation in which the Monte Carlo simulation is to determine the extreme value distribution of a limit state response within a given time duration, i.e., $p_f = \Pr(Y > y)$, where Y is the extreme value of the limit state response; and y is a threshold level. An increasing sequence of intermediate threshold levels, $0 < y_1 < y_2 < \ldots < y_m = y$, can be selected, where m is total number of threshold levels. Denote the first passage probability at the first level y_1 as p_1, and the first passage probability at level $y_i(i = 2,3,\ldots,m)$ conditional on the previous level y_{i-1} as $p_i = \Pr(Y > y_i | Y > y_{i-1})$, the first passage probabilities at different threshold levels are estimated as the product of conditional probabilities:

$$\Pr(Y > y_j) = \prod_{i=1}^{j} p_i (j = 1,2,\ldots,m) \tag{1}$$

Instead of estimating the probabilities $p_i(i = 1,2,\ldots,m)$ for given threshold levels, it is more convenient to adaptively select the threshold levels based on the target probabilities. To determine the first level y_1, N_1 samples are generated from direct MCS. The $(1 - p_1)$-quantile of the sample extremes of the limit state response is determined and chosen as the threshold level y_1. Similarly, the threshold level $y_i(i = 2,3,\ldots,m)$ is selected as the $(1 - p_i)$-quantile of the sample extremes conditional on the samples exceeding the previous level y_{i-1}. Obviously, central to this