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Dynamical analysis of stiffened plates using the compound strip method

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ABSTRACT

The harmonic compound finite strip method has been applied to linear transient vibration analysis of stiffened plates. In this method, eigenfunctions of Bernoulli-Euler beam have been used as the displacement interpolation functions in longitudinal direction, while finite element shape functions have been used for it in transverse direction. The Kirchhoff–Love thin plate theory has been used and the equation of motion of structure is derived from Lagrange's equation of motion. The governing equations have been solved by the mode superposition where step-by-step procedure has been used for the solution of modal equation. The stiffener has been modeled so that it may lie anywhere within the plate strip which helps to increase the flexibility in mesh generation. The formulation is applicable for rectangular plates stiffened with longitudinal and transverse beams and supported on columns. The proposed method is validated through several examples. The strips with free end give erroneous results for non-zero Poisson's ratio.

1. Introduction

Plates stiffened by beams have broad usage in contemporary engineering structures, such as: buildings, bridges, ships, and aircrafts. In these applications stiffened plates are regularly subjected to static and time varying loads. Hence, analysis of stiffened plates under different loading conditions is area of immense interest to researches.

Stiffening of the plate is used to increase its load carrying capacity and to keep structure light weight which makes it cost effective. Because of their high stiffness/mass ratio, these structures are especially vulnerable to dynamic loading.

The analytical solution of the stiffened plates under a time varying loads of arbitrary nature is very difficult, if not impossible. Many numerical methods are developed for analysis of these structures, namely: the finite element method (FEM), the finite strip method (FSM), the boundary element method, the dynamic relaxation method, etc. The FEM is undoubtedly the most versatile and accurate one. Brief review of developments in the analysis of stiffened plates is given by Sapountzakis [1]. It seems that, due to its complexity, a complete understanding of all aspects of stiffened plates behavior is yet to be realized.

In this paper one variation of the well-established FSM is applied. The FSM has been used to solve numerous problems in continuum mechanics [2–4]. This method is semi-analytical finite element procedure. In linear elastic analysis of plates it takes advantage of the orthogonally properties of harmonic functions in the stiffness and mass matrices formulation to yield a block diagonal matrices. In this way two-dimensional problem is decomposed into several one-dimensional sub problems. Unfortunately, the method suffers from a number of drawbacks like mixed boundary conditions, continuous span, internal opening, interior supports and some similar features. These are mostly due to the beam eigenfunctions used as a displacement interpolation function along the longitudinal direction of the strip. Some authors used B-spline functions [5–8] instead of beam eigenfunctions to incorporate stiffeners into arbitrary shaped plates for static and dynamic analysis.

Nevertheless, by using the harmonic functions, Puckett and Gutkowski [9] presented the compound strip method (CSM) for rectangular, and Puckett and Lang [10] for curved plates. They have shown that it is possible to model stiffened plates by direct stiffness method using the FSM with beam eigenfunctions. The method has been applied for free vibration analysis [11,12], statics of bridges and folded plates [13,14], buckling [15] and vibration localization [16].

The CSM presented here is well-suited for calculation of thin rectangular bending plates with arbitrary support conditions. With the CSM it is possible to include stiffness and mass contribution of longitudinal beams, transverse beams and columns into plate.

Stiffened plates subjected to transverse loading suffer in-plane stresses which implies use of in-plane degrees of freedom that again complicates the calculation process. Fortunately, approximative linear elastic analysis can be made by taking into account only out-of plane displacements. Basic idea of the CSM is, like in most numerical methods, to get an economic solution with reasonable accuracy.

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In this paper the CSM has been extended to linear elastic analysis of stiffened plates due to arbitrary immobile dynamic loading.

Extensive analysis has been done by comparing the CSM and the FEM, with four examples provided here. Numerical example of one stiffened plate subjected to dynamical load, as available in literature, has been solved to validate the formulation. We clarified one part of the CSM by showing that this method does not take into account interconnection of beams. Also, well-known problem of plates with free corner has been inspected and it showed that the torsion cannot be adequately approximated with lower order strip.

Results obtained by the proposed method are quite encouraging, especially having in mind that the optimum design of stiffened plate structures demands an effective computational procedure. Simplicity of the CSM approach enables opportunity for wide parametric analysis of stiffened plates in search for optimal placement of stiffeners.

2. Theory

2.1. The finite strip method

Kirchoff–Love thin plate theory implies that only three components of strain exists in plate, and consequently only three components of stress. In this theory only independent variable is deflection of the plate's middle plane – $w(x,y)$.

Typical flat shell lower order (LO2) finite strip of length a, width b and thickness t is presented in Fig. 1.

Displacement function is approximated with truncated series:

$$w(x,y) = \sum_{m=1}^{r} w_m(x) Y_m(y), \tag{1}$$

where r represents number of series terms used in analysis. Parts of approximative function in x direction consist of well-known Hermitian polynomials used as interpolation functions of Bernoulli–Euler beam:

$$w_m(x) = \left(1 - \frac{3x^2}{b^2} + \frac{2x^3}{b^3}\right) w_{0m} + \left(x - \frac{2x^2}{b} + \frac{x^3}{b^2}\right) \varphi_{0m} + \left(\frac{3x^2}{b^2} + \frac{2x^3}{b^3}\right) w_{bm} + \left(-\frac{x^2}{b} + \frac{x^3}{b^2}\right) \varphi_{bm}, \tag{2}$$

where w_{0m}, φ_{0m}, w_{bm}, and φ_{bm} are unknown displacement parameters of nodal lines for mth series term. If we introduce vector $\mathbf{N}$ consisting of Hermitian polynomials N_1, N_2, N_3, and N_4:

$$\mathbf{N}^{\mathrm{T}} = [N_1 \quad N_2 \quad N_3 \quad N_4] \tag{3}$$

and vector of unknown displacement parameters of nodal lines for mth series term

$$\mathbf{q}_m^T = [w_{0m} \quad \varphi_{0m} \quad w_{bm} \quad \varphi_{bm}], \tag{4}$$

we can write Eq. (2) as

$$w_m(x) = \mathbf{N}^{\mathrm{T}} \mathbf{q}_m. \tag{5}$$

This polynomials are providing C^1 continuity which gives reasonable accurate results for bending problems. If more precision is needed, curvature κ can be introduced as the additional degree of freedom in nodal line which yields approximative polynomial of fifth order and provides C^2 continuity. Strip with these two additional degrees of freedom is designated as higher order strip (HO2) [2–4].

Parts of approximative function in y direction, $Y_m(y)$, are chosen as mode shapes of free vibrations of Bernoulli–Euler beam. These functions are the solutions of the differential equation

$$d^4Y(y)/dy^4 - Y(y)(\mu^4/a^4) = 0 \tag{6}$$

and they depend on boundary conditions. All six types of beam's boundary conditions consisting of: simple supported (S), clamped (C) and free (F) are considered here. These well-known eigenfunctions [17] are presented in Table 1.

It should be noted that some of these eigenfunctions require different form because of numerical instability which is introduced for higher modes. Likewise, roots (μ_m) of the characteristic equations which appear when solving Eq. (6), need more accurate values than classical one, presented in Table 1. This numerical instability is caused by subtraction of two nearly equal numbers for higher modes [18]. Having this in mind, some authors suggest usage of different functions which are free of these errors [19,20]. Nevertheless, because of their clear physical meaning, function forms and roots are used in this research and reasonable accurate results are obtained. If one needs large number of series terms included in analysis, function forms presented in [18] are recommended.

2.2. Compound strip method

Typical part of structure suitable for modeling with the CSM is presented in Fig. 2. If we have plate with longitudinal beams (lb), transverse beams (tb) and columns (co), it is possible to model with compound strip. First we have to calculate stiffness and mass properties of line elements relative to the plate's middle plane, and then

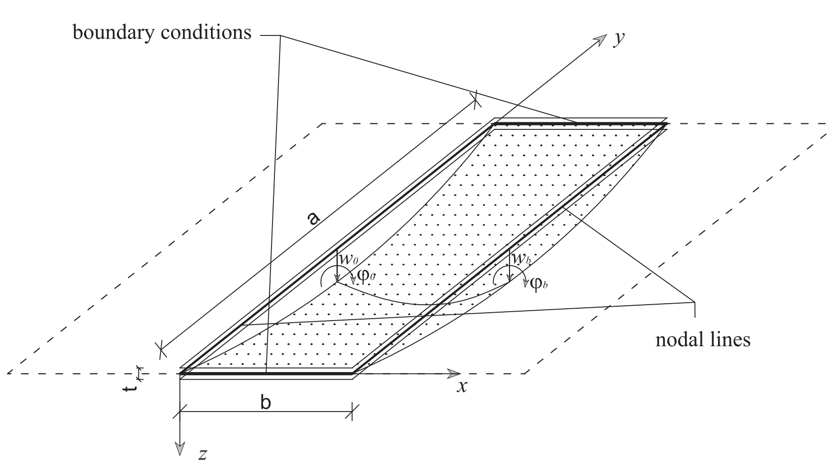

Fig. 1. Flat shell LO2 finite strip – part of the discretized structure.