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Theoretical models to predict the flexural failure of reinforced concrete beams under blast loads

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ABSTRACT

This paper presents two alternative approaches for the study of reinforced concrete beams under blast loads. In the first approach, the beam is modeled by means of Euler–Bernoulli's theory and its elastic–plastic behavior is expressed through a new nonlinear relationship between bending moment and curvature. In the second approach, instead, the beam is idealized as a single degree of freedom system. The effects of strain rate, which are of paramount relevance in blast problems, are taken into consideration by introducing time-variable coefficients into the equations of motion derived from the two models. The latter are employed to assess the time-history of the maximum deflection of a simply supported beam subjected to a uniformly distributed blast load. By comparing the theoretical results with some experimental findings available in literature and with the solution obtained from a commercial finite element software, it is found that the first approach is capable of accurately evaluating the maximum deflection of the beam at failure; on the other hand, the second approach provides a less precise prediction, however it is simpler to implement in practice because it requires less computational effort.

1. Introduction

The effects of explosions on structures can be tremendously dangerous, since they can cause severe damage to buildings and, consequently, loss of lives. Explosions can be ascribed to military events or civilian accidents, like detonations of bombs or other weapons, reactions of certain chemicals, bursts of gas cylinders, and so on. Since the loads due to blast overpressure can be very intense, structural elements should be endowed with enough strength and, above all, ductility to resist such loads. In this paper, the attention is focused on under-reinforced concrete beams, which generally exhibit flexural failure.

Experimental results of structural elements subjected to blast loads are hardly found in literature. This is due not only to national security reasons (the spread of knowledge on this topic is often limited by Countries for defense purposes), but also to the fact that experiments of this kind are costly and difficult to carry out. Fortunately, there are some remarkable exceptions. For instance, Hudson and Darwin [1] damaged several reinforced concrete beams using explosives and, after strengthening some of them with carbon fiber reinforced polymers, examined if the repaired beams exhibited enhanced flexural capacity with respect to the unrepaired ones. Magnusson and Hallgren [2–5] subjected many reinforced concrete beams made of normal or high strength concrete, with or without steel fibers, to air blast loading; they discovered that the beams with a high reinforcement ratio and without steel fibers failed in shear, while those with a low reinforcement ratio failed in flexure. It is important to mention also the experimental works by Remennikov and Kaewunruen [6], Fujikake et al. [7] and Tachibana et al. [8], in which reinforced concrete beams under impact loads were investigated, and the contributions by Alves and Jones [9], Lawver et al. [10] and Nassr et al. [11], who analyzed the behavior of steel members under impact or blast loads; though quite interesting, the results of these studies are not considered in the present paper, since they are not relevant to the scope of this work.

The determination of the dynamic response of a reinforced concrete beam under blast loads is not an easy task, partly because of the complexity in modeling the structural element (considering that the behavior of the beam under such loads is generally nonlinear and that the properties of the materials are functions of the strain rate) and partly because of the difficulty in defining precisely the time variation and the space distribution of the load. Therefore, different simplified methods have been proposed so far. The most elementary and common approach consists in schematizing the beam with a single degree of freedom (SDOF) system [12–15,11]; this approach simplifies both the theoretical formulation of the problem and the calculations, but it usually requires the

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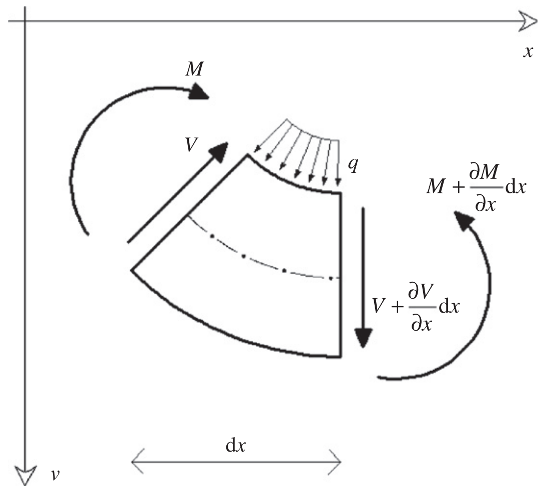

Fig. 1. Internal and external forces acting on an infinitesimal element of the beam.

introduction of empirical formulae and, in addition, it does not provide full information on the beam response. More advanced methods are based on multi-degree of freedom (MDOF) discretization, whereby the beam is divided into finite elements along its length; in the most sophisticated formulations, each element is also subdivided into fibers along its depth, in order to take into account the variation of strain rate over the cross-section [7,16].

In this paper, a continuous model for the reinforced concrete beam (that is referred to as "continuous beam model" in the following) is proposed, in which the inelastic behavior of the beam is represented by a smooth relationship between bending moment and curvature and in which the effects of strain rate are taken into account, as described in Section 2. Furthermore, an idealization of the beam through an equivalent SDOF system is presented in Section 3, where again strain rate effects are considered. The continuous beam model and the SDOF model are employed in Section 4 to derive the time-history of the maximum deflection of a simply supported reinforced concrete beam under blast loads; the validity of these two theoretical approaches is checked by comparing the results they produce with experimental data available in literature and with the solution given by a commercial finite element code. Finally, some concluding remarks are provided in Section 5.

2. Continuous beam model

2.1. Equation of motion of beams under distributed loads

In order to model a reinforced concrete (RC) beam before collapse, Euler–Bernoulli's theory is adopted, which assumes that plane sections remain plane and perpendicular to the beam axis after deformation. An infinitesimal segment of the beam in its deformed configuration is represented in Fig. 1, where v denotes transverse displacements, M is the bending moment, V is the shear force and q is the transverse distributed load. All these quantities are generally functions of both the undeformed axis coordinate x and time t.

Equilibrium conditions require that [17, Section 3.1.1]

$$\frac{\partial^2 M}{\partial x^2} + q = \mu \frac{\partial^2 v}{\partial t^2}, \tag{1}$$

where μ stands for the mass per unit length of the beam. Eq. (1) should be satisfied for any constitutive properties of the beam.

The next step is to specify the nonlinear material behavior of the RC beam. In practical applications, for an under-reinforced concrete beam a bilinear relationship[2] between bending moment and curvature is commonly assumed. Though quite convenient from a computational point of view, this relationship hardly mimics the real behavior of the beam, which rarely exhibits a sharp transition between the elastic and the plastic deformation regimes. Therefore, in this work a smoother relationship between the bending moment M and the curvature θ is introduced, which reads

$$M = \overline{M} \tanh\left(\frac{\overline{K}}{\overline{M}}\theta\right) = -\overline{M} \tanh\left(\frac{\overline{K}}{\overline{M}} \frac{\partial^2 v}{\partial x^2}\right). \tag{2}$$

The hyperbolic tangent function is used in Eq. (2) because it does not present any slope discontinuity and is capable of fitting well a bilinear function by a proper choice of its coefficients. The parameter $\overline{M}$ and $\overline{K}$ appearing in Eq. (2), which represent the equivalent ultimate bending moment and the initial flexural rigidity of the beam respectively, depend on the sectional and constitutive properties of the beam (as illustrated in more detail in Section 2.2). The minus sign in front of the right-hand side term takes into account that a positive bending moment (which, conventionally, causes compression in the top fibers and tension in the bottom ones) produces a negative curvature in the reference system chosen in Fig. 1. It should be observed that the right-hand side term of Eq. (2) is obtained by assuming small deformations and rotations in the beam.

The introduction of a unique relation between bending moment and curvature also allows to use a single equation of motion, without the need to split Eq. (1) into one equation for the elastic range and another one for the elastic–plastic range, as instead required if a bilinear relationship is considered. Moreover, unloading can be ignored in dynamic problems concerning explosions, since the maximum response of the beam to blast loads is usually found before unloading occurs [18].[3] By substituting Eq. (2) into Eq. (1), the following differential equation of motion in the only unknown v is derived:

$$\overline{K}\mathrm{sech}^2\left(\frac{\overline{K}}{\overline{M}}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left[-2\frac{\overline{K}}{\overline{M}}\tanh\left(\frac{\overline{K}}{\overline{M}}\frac{\partial^2 v(x,t)}{\partial x^2}\right)\left(\frac{\partial^3 v(x,t)}{\partial x^3}\right)^2 + \frac{\partial^4 v(x,t)}{\partial x^4}\right] + \mu\frac{\partial^2 v(x,t)}{\partial t^2} = q(x,t). \tag{3}$$

2.2. Neutral axis depth and bending moment at the yield and ultimate states

The parameters $\overline{M}$ and $\overline{K}$ appearing in Eqs. (2) and (3) can be determined from the values of the neutral axis depth and of the bending moment at the yield and ultimate states, as it will be explained in the following.

First of all, the constitutive properties of the materials of which the RC beam is made should be specified. They are extracted from the *fib Bulletin n. 55* [19]. In particular, the uniaxial stress–strain relation of concrete is given by [19, Section 5.1.8.1]

$$\sigma_c = f_{cm}\frac{k \cdot \varepsilon_c/\varepsilon_{c1} - (\varepsilon_c/\varepsilon_{c1})^2}{1 + (k-2)\cdot\varepsilon_c/\varepsilon_{c1}} \quad \text{for } |\varepsilon_c| < |\varepsilon_{c,\lim}|, \tag{4}$$

where σ_c (<0) and ε_c (<0) are concrete compressive stress and strain, respectively, while f_{cm}, ε_{c1}, $\varepsilon_{c,\lim}$ and k are quantities that depend on the concrete grade [19, Tables 5.1–8]. The compressive stress–strain diagram of concrete for a generic concrete grade is shown in Fig. 2a.

[2] A more sophisticated approach would be to adopt a trilinear relationship between bending moment and curvature, which would take into account also the initial state before concrete cracking occurs. Here this initial state is not considered, since the tensile strength of concrete will be neglected, as stated in Section 2.2.

[3] This is the reason why damping has been neglected from the formulation.