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Partial-interaction time dependent behaviour of reinforced concrete beams

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ABSTRACT

When a concrete member is subjected to a load its response is both instantaneous and time dependent. The influence of time dependent deformation is particularly import because it may lead to serviceability failures in structural members where deflections or crack widths are excessive. Current analysis techniques for reinforced concrete members are built around a moment–curvature (M/χ) approach that is based on the assumption of full-interaction (FI), that is, the reinforcement does not slip relative to the concrete which encases it and, consequently, the widening of cracks and their effect on deflection cannot be simulated directly. Hence in order to determine member deflection, empirically derived expressions for the flexural rigidity of a member (EI_{emp}) are required to allow for the tension stiffening associated with cracking. In contrast to this FI M/χ approach, a moment–rotation (M/θ) approach has been developed which allows for slip between the reinforcement and concrete, that is partial-interaction (PI) and which, consequently, obviates the need for the empirically derived flexural rigidities (EI_{emp}). The PI M/θ approach simulates directly, through partial-interaction structural mechanics, the formation and widening of cracks as the reinforcement pulls from the concrete at crack faces and, consequently, automatically allows for tension stiffening. Hence the PI M/θ approach is a useful improvement of the current FI M/χ approach as it quantifies the flexural rigidities associated with tension stiffening which can then be used in standard analysis techniques. It is also shown in this paper that the moment rotation approach can be used to derive flexural rigidities that account for the long term effects of creep and shrinkage as well as predicting the effects of creep and shrinkage on cracks widths and spacings.

1. Introduction

When concrete is subjected to a sustained load, time dependent strains due to creep and shrinkage develop. These creep and shrinkage strains have considerable impact on the performance of structural members, causing increased crack widths and deflections which may result in serviceability failure. The unfavourable nature of time effects on reinforced concrete means it has been an area of research interest for more than 80 years, with much effort devoted to the development of models to predict the changes in concrete material behaviour with time [1–11] and to methods of incorporating these changes into sectional analyses [12–20]. These cited approaches utilise methods of varying complexity to determine the change in concrete material properties with time and, hence, cross sectional behaviour. However in mechanics terms, all of these approaches are based on a moment–curvature (M/χ) analysis technique: in which there is a linear strain profile; and in which there is full interaction (FI), that is, the reinforcement does not slip relative to the concrete so that there is a uni-linear strain profile. These assumptions mean that the techniques are unable to describe crack spacing or widening directly and, therefore, must resort to empirically derived approaches to do so. Thus these approaches ultimately rely on the definition of an effective flexural rigidity (EI_{emp}), which must be defined empirically, to determine member deflection.

In contrast to the FI M/χ approach, a partial interaction (PI) moment–rotation (M/θ) approach for simulating reinforced concrete behaviour under instantaneous loading has been developed by the authors [21–23]; this approach directly simulates what is seen in practice, that is, the formation and widening of cracks using partial-interaction theory [24–30]. In the following paper, the PI M/θ approach is extended to account for the influence of creep and shrinkage. It is first shown how the PI M/θ approach can be applied to a segment of a member to derive the equivalent flexural rigidity of a cross section (EI_{equ}) to allow for tension-stiffening, creep and shrinkage; these equivalent flexural rigidities (EI_{equ}) are a replacement of the empirically derived effective flexural rigidities (EI_{emp}) used in the FI M/χ approach. The equivalent flexural rigidity of a cross section is then used to describe the load deflection behaviour of an entire member through the application of standard analysis techniques. Finally, the approach is

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Nomenclature

A_r	area of tensile reinforcement	t	time
B	bond force	t_0	time at which load is first applied
E_c	elastic modulus of concrete	Δ	slip of the reinforcement in the numerical PI model
E_r	elastic modulus of reinforcing	Δ_1, Δ_2, Δ_3	slip of the reinforcement which define the (τ/Δ) characteristic
EI	flexural rigidity	Δ_c	extension of the concrete from the base line
EI_{emp}	empirically derived effective EI	$\delta\Delta$	change in slip of the reinforcement over a segment
EI_{equ}	equivalent EI	Δ_r	contraction of the reinforcement from the base line
EI_{uncr}	uncracked EI	Δ_{reinf}	slip of the reinforcement form the crack face
L_{bd}	length of concrete prism to FI boundary condition	δ_{top}	deformation of concrete at the top fiber
L_{def}	deformation length	χ	curvature
L_p	perimeter of all reinforcing bars	χ_{sh}	curvature due to shrinkage alone
L_T	total length of reinforcing bar to FI boundary condition	$(\varepsilon_r)_{FI}$	full interaction strain in the reinforcing bar
L_s	segment length for numerical PI analysis	$(\varepsilon_c)_{FI}$	full interaction strain in the concrete
M	moment	ε_{sh}	shrinkage strain
M_{seg}	moment applied to a segment	ε_{pk}	strain corresponding to the peak stress f_c
P_{conc}	force developed in the concrete in compression	ϕ	creep coefficient
$P_{conc\text{-}tens}$	force developed in the concrete in tension	θ	rotation
P_r	force in the reinforcing bar in the PI model	θ_{sh}	rotation due to shrinkage alone
$P_{reinf\text{-}tens}$	force developed in the tension reinforcement	τ	bond stress
$P_{reinf\text{-}comp}$	force developed in the compression reinforcement	τ_{max}	maximum bond stress
$d\Delta/dx$	slip strain	τ_f	frictional bond stress
$(d\Delta/dx)_{FI}$	full interaction slip strain		
f_c	peak concrete stress		

used to predict the behaviour of beams tested by Gilbert and Nejadi [31] under a sustained load and FRP reinforced beams tested by Barris et al. [32] under instantaneous loads, where the PI M/θ approach is used to predict the additional deflections which take place due to shrinkage.

2. Moment–rotation analysis of a segment

The PI M/θ analysis is illustrated in Fig. 1b for a segment of a beam of outline A–A–A–A, of length $2L_{def}$ and of the cross section in Fig. 1a. The segment is symmetrical and symmetrically loaded about E–E so that all deformations can be measured relative to E–E which in effect remains stationary. Prior to any deformations taking place, either as a result of shrinkage or the application of an external load, both the concrete and the reinforcement are of length $2L_{def}$. If a shrinkage strain ε_{sh} is allowed to take place and the concrete were free of any restraint from the reinforcement, a deformation of the concrete of magnitude $\varepsilon_{sh}L_{def}$ from A–A to B–B would take place over each half of the segment A–E. However, due to the presence of internal reinforcement, which in this case is non-symmetrically placed, the concrete is restrained and, hence, the actual deformation of the concrete is from A–A to C–C causing a rotation θ_{sh}. If a constant moment M_{seg} is now applied over the segment, a further rotation takes place such that the total rotation is θ and the deformation is to D–D. By symmetry, the deformations at each end of the segment shown shaded are equal, so that relative to E–E at the mid-length of the segment they produce the same strains or effective strains. Hence it is only necessary to consider one half of the segment which is of length L_{def} in the following analyses. Let us first consider the behaviour of the segment prior to cracking, beginning with the case where the applied moment M_{seg} is zero and, hence, all deformations are the result of shrinkage alone.

3. Segmental analysis prior to cracking

The left hand side of the segment in Fig. 1b is shown in Fig. 2a. The segment has an original length L_{def}; hence prior to any deformation, both the reinforcement and the concrete are of this length. Since any deformation of the reinforcement from this initial length causes a stress to be induced, A–A becomes the baseline for deformations which induce a stress in the reinforcement. Similarly if the concrete were free to shrink without restraint, then it would reduce in length $\varepsilon_{sh}L_{def}$ from A–A to B–B. This shortening would not induce a stress. Hence any deformation of the concrete away from B–B induces a stress in the concrete and, therefore, B–B becomes the baseline for concrete deformations which induce a stress in the concrete.

Prior to the application of any external loads, it is therefore a question of finding a deformation C–C in Fig. 2a, which has a rotation of θ_{sh}, such that for longitudinal equilibrium the moment M_{seg} is zero. To do this, an iterative process is required. The procedure begins by fixing θ_{sh} and guessing the location δ_{top}, thereby, fixing the position of the deformation profile C–C in Fig. 2a. Since the section is uncracked, the deformations can be divided by the deformation length L_{def} to give the strain profiles in Fig. 2b. It needs to be stressed, however, that two strain profiles exist, one for the reinforcement and one for the concrete. Since it has been established that any deformation away from A–A results in a strain to cause a stress in the reinforcement, the deformation from A–A to C–C divided by L_{def} gives the strain profile for the reinforcement, that is, F–F in Fig. 2b. Similarly, since any deformation away from B–B results in a strain to cause a stress in the concrete, the deformation from B–B to C–C divided by L_{def} gives the strain profile G–G in Fig. 2b. It can also be seen in Fig. 2b that these profiles are parallel and located ε_{sh} apart. As the section is uncracked, these strains are real material strains, that is, they would be measured by strain gauges placed on the member. Knowing the distribution of strain in the segment, and because all the strains are real strains, the distribution of stress in Fig. 2c can be determined using any conventional material stress–strain relationship and, hence, the internal forces in Fig. 2d can be determined. If the algebraic sum of these forces is not equal to zero, then the maximum deformation at the top face δ_{top} can be adjusted, thereby shifting the depth of the neutral axis, until equilibrium of the internal forces is achieved, that is they sum to zero. If at this point of longitudinal equilibrium the moment is not zero then θ_{sh} must be adjusted and the analysis