



FE calculations of a deterministic and statistical size effect in concrete under bending within stochastic elasto-plasticity and non-local softening



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ABSTRACT

The numerical FE investigations of a deterministic and statistical size effect in unnotched concrete beams of similar geometry under quasi-static three point bending were performed within elasto-plasticity with non-local softening. The FE analyses were carried out with four different beam sizes. Deterministic calculations were performed with the uniform distribution of a tensile strength. In turn, in statistical calculations, the tensile strength took the form of spatially correlated random fields described by a truncated Gaussian distribution. In order to reduce the number of statistical realizations without losing the calculation correctness, Latin hypercube sampling was applied. The numerical outcomes showed a strong coupled deterministic and stochastic size effect which was compared with the size effect laws by Bazant and by Carpinteri.

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1. Introduction

A size effect phenomenon (nominal strength varies with a characteristic size of a structural member) is an inherent property of the behavior of many engineering materials. In the case of concrete materials, both the nominal structural strength and material brittleness (ratio between the energy consumed during the loading process after and before the stress–strain peak) always decrease with increasing element size under tension [1–3]. When the size and slenderness of the structure are relatively high and the fracture energy is relatively low, the global structural response is brittle. Thus, concrete becomes ductile on a small scale and perfectly brittle on a sufficiently large scale. The results from laboratory tests which are scaled versions of the actual structures cannot be directly transferred to them. The physical understanding of size effects is of major importance for civil engineers who try to extrapolate experimental outcomes at laboratory scale to actual structures of practical size range. Since large structures are strongly beyond the range of testing in laboratories, their design has to rely on a realistic extrapolation of testing results with smaller element sizes.

Two size effects are of a major importance in quasi-brittle and brittle materials: energetic (or deterministic) and statistical (or

stochastic) one (the remaining size effects are [3]: boundary layer effect, diffusion phenomena, hydration heat or phenomena associated with chemical reactions and fractal nature of crack surfaces). According to Bazant and Planas [3] and Bazant [4] the deterministic size effect is caused by the formation of a region of intense strain localization with a certain volume (micro-crack region – called also fracture process zone FPZ) which precedes macro-cracks. The nominal structural strength which is sensitive to the size of FPZ cannot be appropriately estimated in laboratory tests, since it differs for various specimen sizes. Strain localization volume is not negligible to the cross-section dimensions and is large enough to cause significant stress redistribution in the structure and associated energy release. The specimen strength increases with increasing ratio l_c/D (l_c – characteristic length of the micro-structure influencing both the size and spacing of localized zones, D – characteristic structure size). In turn, a statistical (stochastic) effect is caused by the spatial variability/randomness of the local material strength. The first statistical theory was introduced by Weibull [5] (called also the weakest link theory) which postulates that a structure is as strong as its weakest component. The structure fails when its strength is exceeded in the weakest spot, since stress redistribution is not considered. The Weibull's size effect model is a power law and is of particular importance for large structures that fail as soon as a macroscopic fracture initiates in one small material element. It is not however able to account for a spatial correlation between local material properties, does not include any characteristic length of micro-structure (i.e. it ignores a

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deterministic size effect) and it underestimates the effect of small- and intermediate-sizes. Combining the energetic theory with the Weibull statistical theory, a general energetic-statistical theory was developed [6]. The deterministic size effect was obtained for not too large structures and the Weibull statistical size effect was obtained as the asymptotic limit for very large structures. In turn, according to Carpinteri et al. [7–9], the size effect is caused by the multi-fractality of a fracture surface only which increases with a spreading disorder of the material in large structures (stress redistribution and energy release during strain localization and cracking are not considered).

In spite of many experiments exhibiting the noticed size effect in concrete and reinforced concrete elements under different loading types [10–20], the scientifically (physically) based size effect is not taken into account in a practical design of engineering structures, that may contribute to their failure [3,20]. Instead, a purely empirical approach is sometimes considered in building codes which is doomed to yield an incorrect formula since physical foundations are lacking.

The goal of our numerical simulations is to determine in numbers a combined deterministic and statistical size effect in flexural resistance of unnotched beams of a similar geometry under quasi-static three-point bending by using a stochastic enhanced rate-independent continuum concrete model and to compare results with existing size effect laws by Bazant [3,4] and by Carpinteri [7,8]. A finite element method with an elasto-plastic constitutive model using a Rankine's criterion with non-local softening was used which is suitable to capture strain localization under tension. Two-dimensional calculations were performed with four different concrete beam sizes of a similar geometry. Deterministic calculations were performed assuming a constant value of the tensile strength. In turn, statistical analyses were carried out with spatially correlated random fields reflecting the random nature of a local tensile strength. The probability distribution of the tensile strength was described by a truncated Gaussian function. Random fields were generated using a conditional rejection method [21] for correlated random fields. The approximated results were obtained using a Latin hypercube sampling method [22–25] belonging to a group of variance reduced Monte Carlo methods [26]. This approach enables a significant reduction of the number of simulations without losing the accuracy of calculations.

Our calculations are a continuation of our previous simulations of a deterministic and statistical size effect in notched concrete beams within stochastic elasto-plasticity with non-local softening [27]. The difference between the deterministic material strength and mean stochastic strength was practically negligible independently of the beam size and correlation length due to a similar deformation field induced by a notch. The statistical FE results were similar in the case of Latin hypercube sampling with 12 samples and a direct Monte Carlo method with 30 samples.

The combined stochastic-deterministic size effects were simulated by Carmeliet and Hens [28], Frantziskonis [29], Gutierrez and de Borst [30], Gutierrez [31], Vorechovsky [19], Bazant et al. [6,32], Yang and Xu [33] and Bobiński et al. [27]. The most comprehensive combined calculations were performed by Vorechovsky [19] for unnotched concrete specimens under uniaxial tension with a micro-plane material model and crack band approach using Latin hypercube sampling. A squared exponential autocorrelation function with a correlation length of 80 mm was used. His results showed that the strength of many specimens, whose parameters were obtained from random sampling, could be larger than a deterministic one in small specimens in contrast to large specimens which obeyed the weakest link model. The difference between a deterministic material strength and a mean stochastic strength grew with increasing size. The structural strength exhibited a gradual transition from Gaussian distribution to Weibull dis-

tribution at increasing size. When the ratio of autocorrelation length and specimen size decreased, the variation of random field was stronger. In the work by Yang and Xu [33], a heterogeneous cohesive crack model to predict macroscopic strength of materials based on meso-scale random fields of fracture properties was proposed. A concrete notched beam subjected to mixed-mode fracture was modeled. Effects of various important parameters on the crack paths, peak loads, macroscopic ductility and overall reliability (including the variance of random fields, the correlation length, and the shear fracture resistance) were investigated and discussed. It has to be noted that stochastic FE analyses are very difficult due to the lack of experimental results concerning a correlation function and correlation length in engineering materials of a different size.

Our calculations follow the research presented by Vorechovsky [19,34] by using an alternative stochastic approach. In contrast to his simulations: (a) free-supported unnotched concrete beams under bending were analyzed, (b) a more sophisticated regularization technique was used in the softening regime, namely non-local theory, which ensured entirely mesh-independent results with respect to load–displacement diagrams and widths of localized zones (in contrast to the crack band model which provides only mesh-independent load–displacement diagrams), (c) an original method of the random field generation with a different homogeneous anisotropic correlation function was used.

The outline of the present paper is as follows. First, after the introduction (Section 1) and description of size effect laws (Section 2), the employed constitutive elasto-plastic model with non-local softening is summarized (Section 3). The information about the simulation of discrete random fields, finite element discretization and boundary conditions is given in Section 4. The numerical results of a deterministic and stochastic size effect are described and discussed in Section 5. Conclusions are listed in Section 6.

2. Size effect laws

Currently there exist two different theories of size effect in quasi-brittle structures: the energetic-statistical theory [3,4,32] and fractal theory [7–9]. Two size effects laws proposed by Bazant [3,4] (called size effect laws SEL) for geometrically similar structures allow for determining their nominal strength by taking into account the size scale effect. There exist three different types of a deterministic SEL distinguished by Bazant. Type I (Fig. 1a) applies to structures of a positive geometry having no notches or pre-existing cracks for which the maximum load occurs as soon as the FPZ is fully developed and the macroscopic crack can initiate. Type II (Fig. 1b) occurs also for structure with a positive geometry but with notches or large stress-free cracks that grow in a stable manner up to the maximum load. Type III happens in structures of an initially negative geometry where a macro-crack can propagate up to the maximum load (it is very similar to Type II). The requirement of a positive geometry enables to incorporate the weakest link theory by Weibull. The structures obeying the size effect of Type I are sensitive to the material randomness. The nominal strength is strongly affected by the material heterogeneity and decreases with increasing structure characteristic dimension. For Type II, the effect of the material randomness can be ignored. The deterministic size effect of Type I and Type II assumes that the material strength is bound for small sizes by a plasticity limit whereas for large sizes the material follows linear elastic fracture mechanics. The following general analytical formulae for a deterministic size effect predicted by asymptotic matching were proposed by Bazant [3,32]

$$\sigma_N(D) = f_r^\infty \left(1 + \frac{rD_b}{D}\right)^{\frac{1}{n}} \quad (\text{Type 1 size effect law SEL}), \quad (1)$$

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