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Creep analysis of compact cross-sections casted in consecutive stages – Part 1: General method

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ABSTRACT

This paper introduces a general approach to the evaluation of the stress and strain time evolution under long term loading of concrete compact cross-sections casted and/or prestressed in consecutive stages.

The overall cross-section can be made of reinforced concrete, prestressed concrete, structural steel or fiber reinforced polymer parts added at distinct stages of the construction work or during the life of the structural element. Moreover, the cross-section can be prestressed several times both during construction and after gaining the final shape. Therefore, the method herein suggested applies both to new structures casted in consecutive stages and to some problems related to rehabilitation or strengthening of concrete structures.

The physics of this problem leads to a system of Volterra integral equations that, because of the complexity of the creep function usually adopted for concrete, has to be solved by means of a refined step-by-step time integration method which assures high precision to the output.

This general approach and the computer program made with it will be used in a next paper to verify the degree of conformity of the solutions that can be obtained by means of much simpler approaches named algebrized methods.

1. Introduction

The design of structures in which precast main load bearing elements (used as a supporting system for other structural members in order to reduce overall construction time and costs) are combined with cast-in-place concrete (that gives monolithic quality to the structure), provides an economy in the construction as well as high standards of quality while minimizing the time needed to complete the construction. This is the reason why a careful and accurate design of the construction phases is often the key to get a good benefit-cost ratio.

Similarly, structural retrofitting represents an important aspect of the construction industry and its significance is increasing. Retrofitting may occur because of design errors, deficient concrete production or bad execution processes during the construction phases, or because of an earthquake, an accident (such as collisions, fire, explosions), situations involving changes in the structure functionality or because of the development of more demanding code requirements during the service life. The choice of the repair and/or strengthening method depends on the structural behavior objectives, but frequently the methods adopted involve the increase of the reinforced concrete cross-section, that is another construction phase in the life of the structure. This is for instance the case of concrete jacketing of beams and columns (see for instance [1,2]) or strengthening by means of externally bonded FRP (Fiber Reinforced Polymer) systems or steel plates (see for instance [3,4]).

At any rate, the more the construction and/or retrofitting phases are complex, the more computations become complex and cumbersome (see for instance [5,6]), especially when tacking into account creep and shrinkage of concrete under service loads (see for instance [7,8]).

This paper introduces a general approach to the evaluation of the stress and strain time evolution of concrete compact cross-sections casted or prestressed in consecutive stages under long term loading. The overall cross-section can be made of reinforced concrete, prestressed concrete or steel parts added at distinct stages of the construction work. Moreover, the cross-section can be prestressed several times both during construction and after gaining the final shape. Therefore, the method herein suggested applies both to new structures casted in consecutive stages and to some problems related to the rehabilitation or strengthening of concrete structures such as section enlargement and steel or carbon fiber reinforced polymer plate bonding [9].

The physics of this problem leads to a system of Volterra integral equations whose convolution integral (that is the closed-form solution) cannot be determined because of the complexity of the creep function usually adopted for concrete [10,11]. The system of Volterra integral equations is therefore solved by means

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Nomenclature

Notation

A	area
$\underline{A}_g$	matrix of the geometrical properties of the gth piece of concrete
$\underline{B}_a^{(i)}$	stiffness matrix of all the structural steel pieces that are part of the cross-section at time T_i
$\underline{B}_{c_g}^{(i)}$	stiffness matrix of the gth piece of concrete at time T_i
$\underline{B}_{s_g}^{(i)}$	stiffness matrix of all the steel rebars that are part of the gth piece of concrete at time T_i
E	modulus of elasticity
f	total number of structural steel (or steel or carbon fiber reinforced polymer plate) parts of the cross-section
I	second moment of area
$J_g(t_g, \tau)$	creep function of the gth piece of concrete
$\underline{M}(T)$	the vector of the internal forces of the composite cross-section
$\underline{m}_{c_g}(T)$	the vector of the stress resultants (axial force and bending moments measured with respect to the Cartesian axes x and y) inside the gth concrete subsection (i.e. a piece of concrete together with all the rebars and the tendons placed inside it)
NS_m	total number of time steps between time T_m and time T_{m+1}
n	total number of concrete (or other generic linear viscoelastic material) pieces
$\overline{\underline{P}}_g^{(i)}$	vector of the internal actions equivalent to prestressig due to the cables placed inside the gth piece of concrete and tensioned at time T_i
T	time (date, hour, minute etc.)
T_{a_d}	time at which the dth structural steel (or steel or carbon fiber reinforced polymer plate) part of the cross-section becomes effective (is bonded to the section)
T_{q_g}	time at which the gth piece of concrete becomes effective
T_{j_g}	time at which the j_gth prestressing cable (placed inside the gth piece of concrete) is tensioned and grout is injected
$t_{g,i}$	age (days) of the gth piece of concrete at time T_i
x, y, z	Cartesian axes
δ	symbol that represents a change (of any entity involved in the analysis) occurring when the load acting does not change, i.e. a change connected to the delayed behavior of the viscoelastic materials
Δ	symbol that represents a sudden change (of any entity involved in the analysis) due to an instantaneous event (load change, post-tensioning of one or more cables, etc.), i.e. the instantaneous elastic response
ε	strain
ϕ_x, ϕ_y	curvatures (slope of strain diagram)
$\underline{\rho}$	$\underline{\rho} = \lvert 1\ x\ y \rvert^T$
σ	stress
τ	the variable of integration in the Volterra integrals
$\underline{\psi}$	$\underline{\psi} = \lvert \varepsilon_x\ \phi_x\ \phi_y \rvert$

Subscripts

a	structural steel, steel or carbon fiber reinforced polymer plates
c	concrete or another generic linear viscoelastic material
d	the dth structural steel (or steel or carbon fiber reinforced polymer plate) part of the cross-section
g	the gth piece of concrete (or another generic linear viscoelastic material)
i	index of the instantaneous events (load change, post-tensioning of one or more cables, change in the shape of the cross-section, etc.)
j_g	index of the prestressing cables placed inside the gth piece of concrete
k_g	index of the cables placed inside the gth piece of concrete and tensioned at time T_i
s	steel rebars
sh	shrinkage
p	steel tendons
r	index of actual time in the time discretization adopted in the algebraization of the problem
α_g	index of the steel rebars placed in the gth piece of concrete
γ	index of the sampling points (gauss integration formula) in a generic time step

of a refined step-by-step time integration method (based on the techniques suggested by the numerical analysis) which gives rise to an error whose value can be minimized through a suitable choice of the time discretization procedure.

The constitutive law adopted for steel rebars, steel tendons, structural steel (adopted in composite steel-concrete beams and columns), steel and carbon fiber reinforced polymer plates is linear elastic, whereas the constitutive law adopted for each piece of concrete is a generic linear viscoelastic law. When adopting a suitable linear viscoelastic constitutive law (see for instance [12,13]) a piece of concrete can moreover represent an aramid or glass fiber reinforced polymer plate, a glass fiber reinforced polymer poltruded profile, or as a general rule any piece of the cross-section made of a generic linear viscoelastic material.

The main aim of this paper is to get a refined, almost exact solution to all the space-time structural problems already described. In a next paper the output of the computer program written according to this approach will be compared with the outcomes of the much easier, simplified approaches named algebrized methods (mainly the Age Adjusted Effective Modulus method [14–16]) to verify the accuracy the latter when adopted to solve these structural problems.

2. Assumptions and instantaneous response

The assumptions adopted in this numerical model are:

1. The cross-section is made of steel rebars (marked with index s), steel tendons (index p), structural steel, steel or carbon fiber reinforced polymer plates (index a) whose constitutive law is set to be linear elastic (at least under long term service loads), i.e.:

$$\sigma(x,y,T) = \varepsilon(x,y,T) \cdot E \tag{1}$$

and concrete or another generic linear viscoelastic material (index c), whose constitutive law can be written as follows:

$$\varepsilon_c(x,y,t) = \sigma_c(x,y) \cdot J(t,t_o) + \int_{t_o}^{t} d\sigma_c(x,y,\tau) \cdot J(t,\tau) + \varepsilon_{sh}(t) \tag{2}$$