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# Modeling inelastic shear lag in steel box beams

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ABSTRACT

This paper describes a study of inelastic shear lag behavior in steel box beams. Shear lag effects in flanged flexural members are usually recognized as the uneven longitudinal deformation and normal stresses along the flanges. Elastic shear lag behavior has been extensively studied and considered in structural design while the study of inelastic shear lag is limited. Knowing that flanged flexural members likely have plastic deformation at their ultimate limit state, a least-work based method was developed for modeling inelastic shear lag behavior. An effective modulus was formulated and the Poisson's ratio following the theory of plasticity was used in the inelastic shear lag model. The analytical method was verified using laboratory tests of two steel box beams. Comparison with experimental data indicates the proposed variation method can accurately predict the plastic normal strain distribution and the deflection of steel box beams.



## 1. Introduction

The non-uniform normal strains/stresses in the thin-walled flanged flexural members are defined as shear lag [1–3]. Shear lag in elastic structural elements has been studied extensively using analytical methods [2,4–7], finite element analyses [3,8,9] and experimental tests [3,4,10]. Effective flange width used in design guidelines has been derived from the elastic shear lag studies [3,11,12] and implemented in design codes such as American Concrete Institute (ACI) Committee 318 documents (2008), the American Association of State Highway and Transportation Officials (AASHTO) LRFD Specifications (2008), and British Standards (BS 5400). The effective flange width provisions widely adopted in these design codes, which usually require designing structures for ultimate limit states, are based on elastic shear lag analyses. Shear lag effects may not be accurately captured using an elastic analysis when a structural member is loaded into its plastic range of behavior. Meanwhile, capturing the deflections of thin-walled flexural members from the onset of plastic behavior to failure can be critical for modern seismic designs [13,14].

Shear lag behavior in thin-walled flexural members, as described in Moffatt and Dowling [3], Tenchev [8], and Lin and Zhao [15], can be affected by material properties (e.g., Young's modulus and Poisson's ratio). As a material is stressed into its plastic range of behavior, its modulus usually reduces while the Poisson's ratio increases. For example, the elastic Poisson's ratio of steel is usually 0.3 while concrete 0.15–0.2. However, Poisson's ratio will dramatically increase near 0.5 for steel in the plastic range [16,17]. Similarly, Brandtzaeg's test, as documented by Klieger and Lamond [18], has shown significant increases in Poisson's ratio of concrete at relatively high compressive stresses. Hence, the prediction of shear lag behavior, especially after the onset of plasticity (e.g. first yielding in steel) needs to consider these nonlinear material properties.

An analytical model was established to model inelastic shear lag behavior in steel box beams in this study. The analytical model consists of a least-work based solution with an effective modulus and a plastic Poisson's ratio. Comparison of the measured and the predicted inelastic shear lag behavior indicated that the proposed method can accurately capture the inelastic normal flange strains and plastic member deflections.

## 2. Review of elastic shear lag model

The development of the least-work based method is demonstrated using a generalized box beam as shown in Fig. 1 though the same methodology can be implemented in other thin-walled flexural members such as non-rectangular shear walls. The least-work solution to the shear lag problems in a box beam provides closed-form solutions for normal strains/stresses, which facilitates the study of the major factors for shear lag behavior, including both material properties and the geometric properties. Rather than a single binomial function used in most previous solutions [2,4], a $2m$-degree polynomial (the summation of $m$ terms of binomials with even exponents) was used to accurately describe the normal displacement along the flange [7,19]. The longitudinal flange displacements, $u(x, y)$, was defined as a combination of the

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**Nomenclature**

| | |
|---|---|
| $A_f$, $A_w$, $A$ | area of the flange, the web and the cross section of a box girder |
| $A_s$ | area of stiffeners on the compression flange |
| $b_e$ | effective flange width |
| $E$, $G$, $E_t$ | Young's modulus, shear modulus and tangent modulus |
| $E_e$, $G_e$, $v$ | effective modulus, effective shear modulus, and plastic Poisson's ratio |
| $h$, $L$ | the height and the span of the box girder |
| $I_e$ | the effective moment of inertia of the box girder |
| $I_s$, $I_w$, $I$ | the moment of inertia of the flanges, the webs and the box girder |
| $M_y$, $M(x)$ | the yield bending moment and bending moment |
| $P$, $a$ | point load and the position of the load |
| $Q_w$ | the first moment of the web |
| $u(x, y)$ | normal flange displacement of a box girder |
| $U_i(x)$ | the parameter for the $i$th term in the Taylor series shape function |
| $w$ | the deflection of a box girder |
| $\sigma_x$, $\varepsilon_x$; $\sigma_{\max}$ | normal stress, normal strain and maximum stress of a box girder |
| $\varepsilon_y$ | yield normal strain |
| $\varphi(x)$ | the curvature of the girder |
| $\kappa$ | effective section factor |

contributions from both flexural deformation ($\varphi(x)$) and the shear lag effect,

$$u(x,y) = \pm h\left[\varphi(x) + \sum_{i=2}^{2m}\left(1 - \frac{y^i}{b^i}\right)U_i(x)\right] + U_a(x), \quad (1)$$

where $h$ is the distance of the center of the flange plates to the neutral axis (e.g., $-h_t$ and $+h_b$ are for the top and the bottom flange, respectively), $b$ is the width of the flanges (i.e., $b_1$, $b_2$, $b_3$ for the bottom, top and cantilever flange as illustrated in Fig. 1), $U_i(x)$ describes the contribution of the shear lag related displacement function $(1 - y^i/b^i)$, and $U_a(x)$ was used to correctly locate the neutral axes after Ni [20]. Note that the earlier least-work solutions developed by Reissner [2] truncate the high-order polynomials shown in Eq. (1) down to the first term, thus is limited in its accuracy. With this high-order polynomials, variations in the flange displacements along the member, as observed and documented in the literature [9,21], can be closely represented.

With detailed derivation shown elsewhere [7,19], a group of dependent differential equations could be obtained by minimizing the total potential energy of the member and by employing the member boundary conditions. The unknown functions, $U_i(x)$, were determined by solving an Eigenvalue problem. The normal strains in the flange were then obtained as the first derivative of the flange displacement with respect to the $x$-axis (along the member). For example, the strain in the flange of a simply supported box beam with a span $L$ is:

$$\varepsilon_x(y) = -\frac{Mh}{EI} + \frac{h}{G}\sum_{i=2}^{2m}\left(1 - \frac{y^i}{b^i} - \frac{i}{i+1}\frac{I_s}{I} - \frac{i}{i+1}\frac{Q_w}{Ah}\right)U_i'(x), \quad (2)$$

where $E$ and $G$ are Young's modulus and shear modulus, respectively; $M$ is the bending moment along the beam; $Q_w$ is the first moment of the webs with respect to the neutral axis and $A$ is the total area of the box section. $I_s = 2t_2b_2h_t^2 + 2t_3b_3h_t^2 + 2t_1b_1h_b^2$ is the moment of inertia of the flange plates and $I_w = 2t_w(h_1^2 + h_2^2)/3$ is the moment on inertia of the web plates, and hence the total moment of inertia is $I = I_s + I_w$. The strain solution in Eq. (2) indicates that the flange normal strain consists of bending effects and the shear lag contribution, which is related to the shear modulus ($G$) and other geometric properties of the member. Normal stresses can be calculated following the Hooke's law for linear elastic materials.

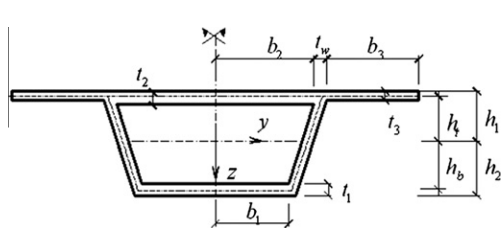


**Fig. 1.** Schematics of a box girder.

As the applied load increases such that the flanges start yielding, shear lag effects on the flange normal strains become more outstanding as shown later in Fig. 6. The increased shear lag effect is mainly attributed to a reduced modulus and an increased Poisson's ratio.

## 3. Modeling inelastic shear lag

The flange of thin-walled flexural member such as a steel box beam (Fig. 2a) partially enters into plastic stage after the first yielding at the critical section (mid-span for simply supported beams) as illustrated in Fig. 2b. In addition, the plastic region propagates from the critical section into the rest of the beam as the external loads (or moments) increases. In this case, shear lag is affected partial yielding along the beam and partial yielding cross a section as illustrated in Fig. 3 and shown later in the experimental tests.

Most existing studies used finite element methods (FEM) for modeling inelastic shear lag effects in steel–concrete composite girders. For example, Amadio and Fragiacomo [22] used shell elements in ABAQUS® in a parametric study of effective flange width for steel composite girders. It was concluded that the shear lag effects (in terms of effective flange widths) in the plastic zone could be affected by several parameters, including the amount of

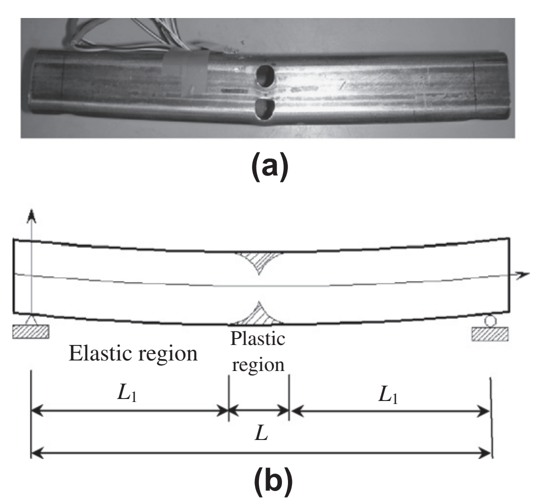


**Fig. 2.** The elastic and plastic regions in a box beam.