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Frequency characteristics of railway bridge response to moving trains with consideration of train mass

Yong Lu [a,c,*], Lei Mao [a,c], Peter Woodward [b,c]

[a] Institute for Infrastructure and Environment, School of Engineering, The University of Edinburgh, The Kings Buildings, Edinburgh EH9 3JL, UK
[b] Institute for Infrastructure and Environment, School of Built Environment, Heriot-Watt University, Edinburgh, UK
[c] Joint Research Institute for Civil and Environmental Eng., Edinburgh Research Partnership in Engineering and Mathematics, UK

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ABSTRACT

The dynamic response of railway bridges is known to be influenced by a combination of factors including the bridge natural frequency, train speed, and bridge and carriage lengths. However, the intrinsic relationships among these parameters have seldom been elaborated in common dynamics terms so as to enable more effective implementation in practice. This paper attempts to approach this classic problem from a frequency perspective, by investigating into the frequency characteristics in the bridge response as well as in the moving trainloads. In particular, the significance of the so-called "driving" and "dominant" frequencies arising from the moving load is examined. Based on numerical results and a securitization using a generalised trainload pattern, it is demonstrated that the primary frequency contents in the trainload, and consequently in the dynamic response of the bridge, is largely governed by the bridge-to-carriage length ratio. Namely, for short bridges (with a length ratio below the order of 1.5), well-distributed frequency peaks occur at a number of dominant frequencies, whereas for longer bridges the main frequency peak tends to concentrate towards the lowest dominant frequency. Such a characteristic affects directly the resonance condition and resonance speeds for bridges of different length categories, and this observation echoes well the predictions of the resonance severity using a so-called *Z*-factor. For the special case of bridge response under a single carriage/vehicle, the influence of the carriage mass is examined in association with the concept of critical speed, and the abnormal acceleration spikes that could occur when the vehicle moves at the critical speed is highlighted.

1. Introduction

The dynamic response of railway bridges is complicated due to the involvement of moving loads and moving masses. Comparing to road traffic, the trainload excitation is characterised by a unique pattern of frequency spectrum, which directly affects the dynamic response of the bridge. Moreover, the dynamic properties of railway bridges, especially the natural frequencies of small- to medium-size bridges, can be altered significantly due to moving carriage masses.

Numerous publications exist in the literature regarding bridge dynamic response and the train–bridge interactions. It is well recognised that the dynamic response of a railway bridge is influenced by a combination of factors, chiefly the bridge natural frequency, train speed, and bridge and carriage lengths. However, the intrinsic relationships among these parameters have generally been implicitly expressed through dynamic formulations, whereas specific guides for their application in practice are lacked. For example, it is not straightforward to implement a general recommendation that resonance could take place under certain normalised speeds, without specific information with regard to the resonance severity and an understanding of the trend of variations.

A seemingly effective way of approaching this subject is to resort to the frequency analysis of the trainload in conjunction with the frequency characteristics of the responding system. However, studies stemming from a frequency perspective are still limited. Those that fall into this category may be loosely divided into two groups, one concerns the variation of the natural frequencies of the responding bridge during the passage of a laden train/vehicle (e.g. [1–5]), and the other deals with the frequency contents in the trainload excitation and their general effect in the bridge response (e.g., [5–8]). In particular, it has been demonstrated that, in addition to the resonant frequencies, the primary frequencies in the bridge response may be attributable to (a) the so-called "driving frequencies" associated with the duration of a vehicle crossing the bridge [8], and (b) the so-called "dominant frequencies" arising from the repeated loads (hence are related to the time

* Corresponding author at: Institute for Infrastructure and Environment, School of Engineering, The University of Edinburgh, The Kings Buildings, Edinburgh EH9 3JL, UK. Tel.: +44 131 6519052; fax: +44 131 6506781.
E-mail address: yong.lu@ed.ac.uk (Y. Lu).

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interval between two consecutive carriage loads) (e.g., [7,9]). Despite the identification of these frequency factors, the understanding of their influence on the bridge response remains to be rather general.

The present paper aims to provide a comprehensive evaluation of the frequency characteristics of a railway bridge response under trainload, paying special attention to examining the significance and the variation trend of key frequency components in the response arising from the trainload, namely the driving frequencies and dominant frequencies mentioned above. To incorporate the influence of the moving mass, the analysis is carried out using a finite element model, in which a moving vehicle is simulated with a moving mass block which is coupled with the bridge via surface contact. For simplicity while withholding the primary frequency characteristics, the vehicle dynamics and track irregularities are not considered.

It is particularly worth noting that the relative length of the bridge with respect to the length of the carriage is found to be a governing factor determining the characteristic patterns and hence the frequency contents in the trainload excitation, and therefore this length ratio is employed in the classification of the frequency response characteristics. Following the establishment of the frequency characteristics, the bridge resonance effect is evaluated through a series of parametric calculations. The observations on the resonance phenomenon are then correlated with a newly proposed resonance severity factor, called the *Z*-factor [10], to provide a complete framework for the understanding as well as quantification of the bridge resonance under moving trains.

2. Background theories

2.1. Natural frequency of bridge–moving train system

When a train moves on a bridge, the frequencies of the bridge will be affected due to the effects of train mass coupled with the bridge through the suspension systems. When the train mass is large with respect to the mass of the bridge, such effect can become significant.

The natural frequencies of the bridge during the passage of a train (or a single vehicle as a specialised case) may be established on the basis of the dynamic equation for the bridge coupled with the moving object, as follows:

$$m_b \frac{\partial^2 w_b}{\partial t^2} + EI \frac{\partial^4 w_b}{\partial x^4} + c_b \frac{\partial w_b}{\partial t} = P(x, t) \tag{1}$$

where EI, m_b, c_b are the flexural stiffness, mass per unit length and damping coefficient of the bridge, w_b is the bridge vertical displacement, and $P(x, t)$ is the interacting force between the vehicle and the bridge.

The interacting force with the ith wheel–axle set may be expressed as [2]:

$$P_i(x, t) = \delta[x - (Vt - a_i)] \left(P_{0,i} - m_c \frac{\partial^2 w_b}{\partial t^2} + c_c \dot{w}_i + k_c w_i \right) \tag{2}$$

where δ is the Dirac delta function, $P_{0,i}$ is the static weight borne by the ith wheel–axle set, a_i is the distance between the first and ith wheel–axle sets, w_i is the displacement within the suspension spring, m_c denotes the effective mass that may be attributed to a wheel–axle set, c_c, k_c are spring damping and stiffness of the vehicle's suspension system, respectively.

The solution of the motion equation can be obtained by modal superposition. Denoting the nth mode shape as $\phi_n(x)$ and the generalised modal coordinate as $q_{bn}(t)$,

$$w_b(x, t) = \sum_n \phi_n(x) q_{bn}(t) \tag{3}$$

For a simply supported bridge (beam), the mode shapes may be expressed in a sinusoidal form, thus:

$$w_b(x, t) = \sum_n \sin \frac{n\pi x}{L_b} q_{bn}(t) \tag{4}$$

where L_b is the bridge length.

Substituting Eqs. (2) and (4) into Eqs. (1), multiplying both sides with $\sin(n\pi x/L_b)$, and then integrating with respect to L_b yields:

$$\ddot{q}_{bn}(t) + 2\xi_{bn}\omega_{bn}\dot{q}_{bn}(t) + \omega_{bn}^2 q_{bn}(t) = \frac{2}{m_b L_b} P_{bn}(t) \tag{5}$$

where ω_{bn} is the nth natural frequency, ξ_{bn} is the corresponding damping ratio, $P_{bn}(t)$ is the generalised modal force and may be expressed [1] as:

$$P_{bn}(t) = \sum_{i=K}^{M} \sin \frac{n\pi(\upsilon t - a_i)}{L_b} \left[P_{0,i} - m_s \sum_{k=1}^{\infty} \sin \frac{k\pi(\upsilon t - a_i)}{L_b} \ddot{q}_{bk}(t) + c_c \dot{w}_i + k_c w_i \right] \tag{6}$$

A numerical integration method, such as the Wilson-θ method, can be employed to obtain the bridge natural frequencies at each time step. The results will allow the variation of the bridge frequency to be plotted against time or the position of the moving train.

2.2. Driving frequencies

The so-called "driving frequency" [8] is associated with the inverse of the time duration a vehicle crosses the bridge. Specialising Eq. (2) into a single moving load,

$$P(x, t) = f_c(t)\delta(x - Vt) \tag{7}$$

where $f_c(t)$ is the sum of the vehicle weight and the dynamic force of the suspension system,

$$f_c(t) = -m_c g + k_c(w_c - w_b) \tag{8}$$

where k_c is the stiffness between moving mass and the bridge, w_c and w_b are the dynamic deflections of the moving mass and the bridge, respectively.

Substituting Eq. (7) into Eq. (5), and ignoring the damping term yields:

$$\ddot{q}_{bn} + \omega_{bn}^2 q_{bn} = \frac{f_c(t) \int_0^L \delta(x - Vt)\phi_n(x) \mathrm{d}x}{m_b \int_0^L \phi_n^2(x) \mathrm{d}x} \tag{9}$$

Combining with the motion equation of the moving mass, and substituting Eq. (8), Eq. (9) may be re-written as [8]:

$$\ddot{q}_{bn} + \omega_{bn}^2 q_{bn} + \frac{2\omega_c^2 m_c}{m_b L_b} \sin \frac{n\pi Vt}{L_b} \sum_j \sin \frac{j\pi Vt}{L_b} q_{bj} - \frac{2\omega_c^2 m_c}{m_b L_b} \times \sin \frac{n\pi Vt}{L_b} q_c = \frac{-2m_c g}{m_b L_b} \sin \frac{n\pi Vt}{L_b} \tag{10}$$

If the mass of the passing vehicle is much less than that of the bridge and hence may be ignored, the above equation reduces to:

$$\ddot{q}_{bn} + \omega_{bn}^2 q_{bn} = \frac{-2m_c g}{m_b L_b} \sin \frac{n\pi Vt}{L_b} \tag{11}$$

Assuming a zero initial condition, the solution to the above equation may be obtained as:

$$q_{bn}(t) = \frac{\Delta}{1 - S_n^2} \left[\sin \frac{n\pi Vt}{L_b} - S_n \sin(\omega_{bn} t) \right] \tag{12}$$