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A mathematical model for assessment of material requirements for cable supported bridges: Implications for conceptual design

W.J. Lewis *

School of Engineering, University of Warwick, Coventry CV4 7AL, UK

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ABSTRACT

Recent technological developments have led to improvements in the strengths of materials, such as the steel and wire ropes used in the construction of cable supported bridges. This, combined with technological advancements in construction, has encouraged the design of structures with increasing spans, leaving the question of material and environmental costs behind. This paper presents a refined mathematical model for the assessment of relative material costs of the supporting structures for cable-stayed and cable suspension bridges. The proposed model is more accurate than the ones published to date in that it includes the self weight of the cables and the pylons. Comparisons of material requirements for each type of bridge are carried out across a range of span/dip ratios. The basis of comparison is the assumption that each structure is made of the same material (steel) and carries an identical design load, *q*, exerted by the deck. Calculations are confined to a centre span of a three-span bridge, with the size of the span ranging from 500 m to 3000 m. Results show that the optimum span/dip ratio, which minimises material usage, is 3 for a cable-stayed (harp type) bridge, and 5 for a suspension structure. The inclusion of the self weight of cable in the analysis imposes limits on either the span, or span/dip ratio. This effect is quantified and discussed with reference to the longest cable-supported bridges in the world completed to date and planned in the future.

1. Background

Over the years, a number of studies related to the assessment of the volume of material and material costs in cable-stayed and suspension bridges have been produced. A relatively simple model used by French [1], which excluded the self weight of cables and pylons, demonstrated that, with the cost of the cable material twice that of the pylons, the optimum span/dip ratio for the suspension bridges was 9:1. This prediction was based on the allowable stresses in the cables of 600 N/mm^2, and 120 N/mm^2 in the pylons, which values were significantly lower than the up-to-date strengths of 700 N/mm^{-2} and 160 N/mm^2, respectively, as used by Gimsing [2] and the author of this paper.

The model proposed by Gimsing [2], included the self weight of pylons, as this was viewed as important in the final assessment of the 'lightness' of the structure, but excluded the self weight of the cables. Surprisingly, it also excluded the additional weight of the deck required in the cable-stayed bridge to resist the substantial membrane forces that develop there. Based on these assumptions, the model predicted an optimum span/dip ratio for both suspension and cable-stayed (fan type) bridges with the main span of 500 m to be $\sim$6.6. This was based on material costs, assuming that the unit cost of steel in the pylons was the same for both systems, but the ratios of the unit price of cable to pylon were different: 1.75 in the case of the suspension bridge, and 2.5 for the cable-stayed structure. This optimum span/dip ratio was unchanged for the suspension bridge when the main span was doubled, i.e., equal to 1000 m.

Earlier work by Podolny and Scalzi [3] stated that the most economical span/dip ratio for the cable-stayed bridges was 5:1, and 8:1 for the suspension type. It reported on the work of Leonhardt and Zellner [4], which produced a modification factor on the material volume used by the cable when the cable weight was included. Taking the span/dip ratio of 9:1 for the suspension bridge, a 5:1 for the cable-stayed one, and a central span of 3280 ft. (1000 m), the proposed modification increased the cable steel requirements for the suspension bridge by 17%, but only by 5% for the cable-stayed structure.

The work quoted above highlights the issue of scale. Parsons [5] showed that on the basis of an approximate relationship between the cost per unit area of roadway and the span, suspension bridges were more economic for spans above 600 m (the height of the pylons was not given). He stressed the fact that the span of a suspension bridge was limited only by the tensile strength of the cable and this prime structural element is inherently stable, while the span of a cable-stayed structure is limited by the compressive strength of the deck which is inherently unstable.

* Tel.: +44 24 76 523 138.
E-mail address: W.J.Lewis@warwick.ac.uk

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More recently, Croll [6] offered a simple analysis of the relative usage of the material by cable-stayed (harp type) and suspension bridges, respectively. In common with Gimsing [2] and French [1], the calculation for the volume of the material was based on the design principle that the cross-section of any load-carrying member should not be stressed beyond an assumed value of working stress. The analysis ignored the self weight of the cables and the pylons. Surprisingly, in the initial model, no distinction was made between the tensile strength of the cables and the pylons, and simply one value was used for both. The modification of the model, following contributions from Dalton et al. [7], included not only material usage, but also material cost. The calculated material volumes were factored using a compound material and cost parameter, β, expressed as a ratio of tensile to compressive stresses, further multiplied by a ratio of unit costs of cable to pylon. The factor β ranged between 1 and 5. After this modification, the results showed the suspension bridges to be more cost efficient than the cable-stayed ones, for span/dip ratio greater than 4. They also showed an optimum span/dip ratio for a cable-stayed bridge to be between 2 and 3 and, for a suspension structure, between 4 and 7 (depending on the β factor).

In view of the inconsistent and conflicting information produced to date, a more rigorous analysis of material usage (including material cost) is needed in the design of cable supported bridges. This paper addresses this problem by examining the subject more closely and presenting an analysis that is as close to reality as possible.

2. Suspension bridge

2.1. General

Fig. 1 shows the basic geometry of a suspension cable bridge in which L is the centre span of the bridge and h is the height of towers above the deck. The distribution of forces in the main structural elements is shown in Fig. 1a.

The prediction of material usage is based on the centre span. The general assumptions are as follows:

(i) The bridge is subjected to a uniformly distributed design deck load, q, the weight of the cables and the pylons.
(ii) The shape adopted by the suspension cables is assumed to be a parabola. This shape corresponds to the case of a uniformly distributed load from the deck, q, and follows the usual assumption that hanger and cable weights are negligible compared to q.
(iii) The hangers form a uniform 'curtain' suspended from the cables and stressed by q. It is shown later that the stress due to the self weight of hangers is negligible. Hence, the amount of material used by them is simply proportional to the area under the parabolic cable.
(iv) The cross-section of the suspension cable is calculated by dividing the maximum tension force in the cable by an assumed constant value of working (tensile) stress, σ_t; the product of the cross-section area and the length of the cable gives the volume of the material required.
(v) Each pylon is assumed to carry a half of the deck weight, $qL/2$, the self weight of the cables and their own weight; their cross-section area varies with height in such a manner as to ensure constant stress.

2.2. Calculation of the volume of material used by hangers

The hangers, modelled as a 'curtain' of individual strands, have thickness, t. If ρ_h is the density of the hanger and $T_{hng}(z)$ is the tensile stress, the equation of vertical equilibrium of the hanger element, $tdxdz$, (Fig. 1a) is

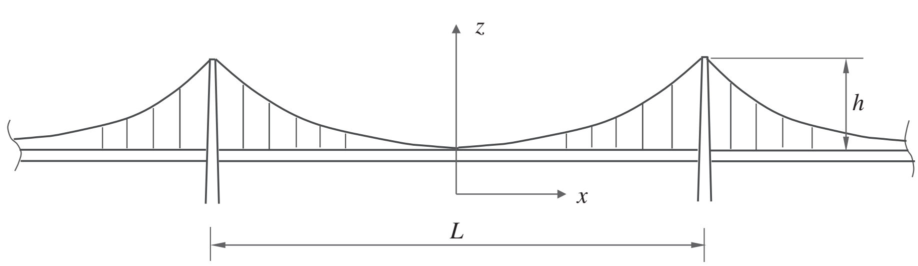

Fig. 1. Basic geometry of a suspension cable bridge.

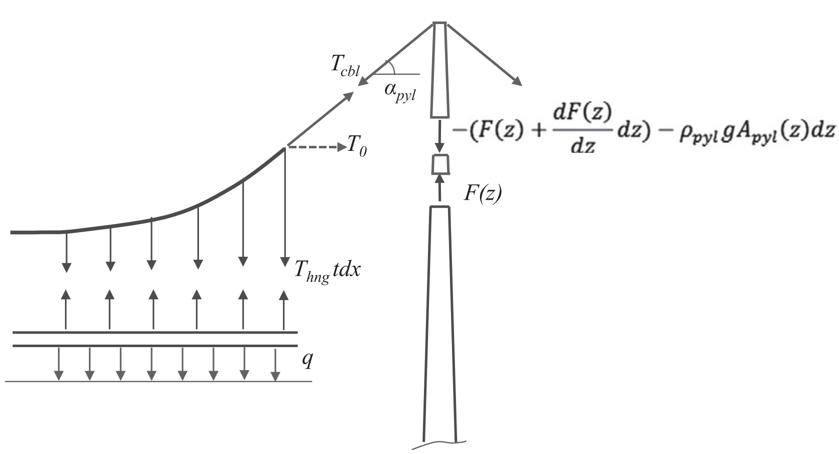

Fig. 1a. Diagrammatic representation of forces in the main structural elements of the bridge.